CHAPTER 11

Areas of Polygons and Circles

Targeted TEKS
G.10(B), G.11(A), G.11(B), G.12(C)

Mathematical Processes
G.1(A), G.1(B), G.1(C), G.1(D), G.1(E), G.1(F), G.1(G)

THEN

You learned about circles and angles within circles.

NOW

In this chapter, you will:

- Find areas of polygons.
- Solve problems involving areas and sectors of circles.
- Find scale factors using similar figures.

WHY

AGRICULTURE NASA's Landsat Satellites credit center-pivot irrigation with the formation of crop circles in the Texas panhandle. Farmers use area to determine where the sprinklers should be placed.

Go Online!
connectED.mcgraw-hill.com

Vocabulary is important to learning the key concepts in this chapter. Find all the terms with animations, English pronunciations, and translations into 13 languages in the eGlossary in ConnectED.

ALEKS | The Geometer's Sketchpad | Vocabulary | Tutor | Tools | Calculator Resources | Check | Worksheets | Watch

Get Ready for the Chapter

Go Online! Take the Chapter Readiness Quiz online as another option

Diagnose Readiness | **You have two options for checking prerequisite skills.**

Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

The area and width of a rectangle are given. Find the length of the rectangle.

1. $A = 25, w = 5$

2. $A = 42, w = 6$

3. $A = 280, w = 14$

4. $A = 360, w = 60$

5. **GARDENS** Molly planted a garden with a length of 72 feet. If she bought enough fertilizer to cover 792 square feet, what width should she make the garden?

Evaluate each expression if $a = 9$, $b = 10$, $c = 12$, and $d = 13$.

6. $\frac{1}{2}a(b + c)$

7. $\frac{1}{2}(ab + cd)$

8. $\frac{1}{2}(a + bd)$

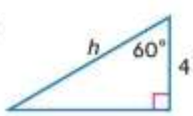
9. $\frac{1}{2}cd$

10. $\frac{1}{2}(ab + c)$

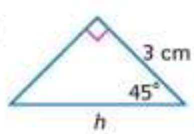
11. $\frac{1}{2}(a + d)$

Find *h* in each triangle.

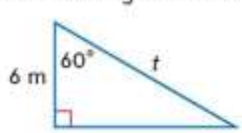
12.



13.



14. **LOOKOUT** The lookout on a pirate ship slides down a rope from the top of the mast 6 meters above the water. He can see the land at a 60° angle. How far does he slide?



QuickReview

Example 1

The area of a rectangle is 64 square units and the width is 4 units. Find the length.

$A = \ell w$ Area of rectangle

$64 = \ell(4)$ Substitution

$16 = \ell$ Divide each side by 4.

The length is 16 units.

Example 2

Evaluate $\frac{1}{2}x(2x + 3y)$ for $x = 4$ and $y = 12$.

$\frac{1}{2}x(2x + 3y) = \frac{1}{2}(4)[2(4) + 3(12)]$ Substitution

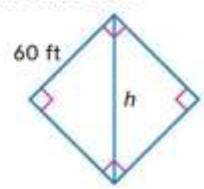
$= 2(8 + 36)$ Multiply.

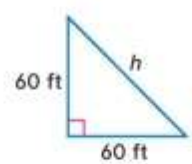
$= 2(44)$ Add.

$= 88$ Multiply.

Example 3

Find the value of *h*.





Since *h* is the hypotenuse of the triangle, the triangle can be redrawn as shown.

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times the length of a leg.

$h = (\sqrt{2})60$

≈ 84.85

So, *h* is approximately 84.85 feet.

Get Started on the Chapter

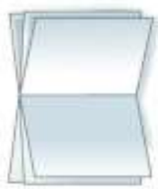
Go Online! for Vocabulary Review Games and key vocabulary in 13 languages

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 11. To get ready, identify important terms and organize your resources. Working with a partner can be helpful as you prepare and as you read the chapter.

FOLDABLES® StudyOrganizer

Areas of Polygons and Circles Make this Foldable to help you organize your Chapter 11 notes about areas of polygons and circles. Begin with three sheets of notebook paper.

1 **Stack** three sheets of paper and fold them in half, lengthwise.



2 **Staple** the papers together one inch from the top fold.

3 **Cut** the top sheet two inches from the top fold and each following sheet one inch longer than the previous sheet.

4 **Label** as shown.

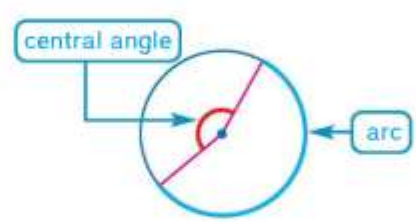
NewVocabulary

English		Español
base of a parallelogram	p. 779	base de un paralelogramo
height of a parallelogram	p. 779	altura de un paralelogramo
base of a triangle	p. 781	base de un triángulo
height of a triangle	p. 781	altura de un triángulo
height of a trapezoid	p. 789	altura de un trapecio
sector of a circle	p. 799	sector circular
center of a regular polygon	p. 807	centro de un polígono regular
radius of a regular polygon	p. 807	radio de un polígono regular
apothem	p. 806	apotema
central angle of a regular polygon	p. 807	ángulo central de un polígono regular

ReviewVocabulary

arc arco a part of a circle that is defined by two endpoints

central angle ángulo central an angle that intersects a circle in two points and has its vertex at the center of the circle



diagonal diagonal a segment that connects nonconsecutive vertices of a polygon

LESSON 11-1

Areas of Parallelograms and Triangles

Then

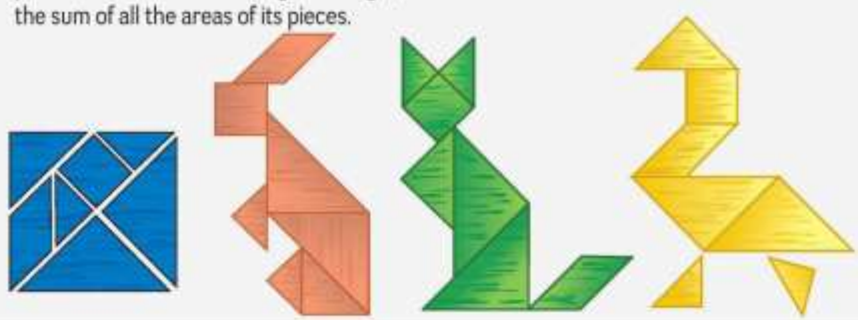
- You found areas of rectangles and squares.

Now

- 1 Find perimeters and areas of parallelograms.
- 2 Find perimeters and areas of triangles.

Why?

- A tangram is an ancient Chinese puzzle that can be rearranged to form different images, such as the animals shown. The area of the puzzle, before and after being rearranged, remains the same. It is the sum of all the areas of its pieces.



Targeted TEKS

G.10(B) Determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.

Mathematical Processes

G.1(B) Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

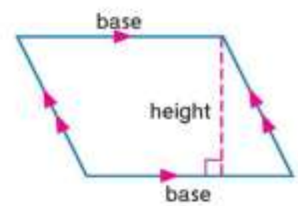
G.1(E) Create and use representations to organize, record, and communicate mathematical ideas.

NewVocabulary

base of a parallelogram
height of a parallelogram
base of a triangle
height of a triangle

1 Areas of Parallelograms

In Lesson 6-2, you learned that a *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called the **base of a parallelogram**. The **height of a parallelogram** is the perpendicular distance between any two parallel bases.



You can use the following postulate to develop the formula for the area of a parallelogram.

Postulate 11.1 Area Addition Postulate

The area of a region is the sum of the areas of its nonoverlapping parts.

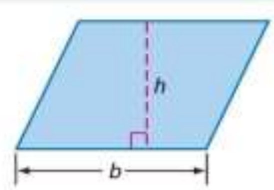
In the figures below, a right triangle is cut off from one side of a parallelogram and translated to the other side as shown to form a rectangle with the same base and height.

Recall from Lesson 1-6 that the area of a rectangle is the product of its base and height. By the Area Addition Postulate, a parallelogram with base b and height h has the same area as a rectangle with base b and height h.

KeyConcept Area of a Parallelogram

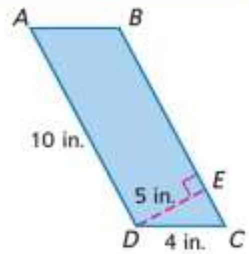
Words The area A of a parallelogram is the product of a base b and its corresponding height h.

Symbols $A = bh$



Example 1 Perimeter and Area of a Parallelogram

Find the perimeter and area of ▱*ABCD*.



Perimeter

Since opposite sides of a parallelogram are congruent, $\overline{AB} \cong \overline{DC}$ and $\overline{BC} \cong \overline{AD}$. So $AB = 4$ inches and $BC = 10$ inches.

Perimeter of ▱$ABCD = AB + BC + DC + AD$
$= 4 + 10 + 4 + 10$ or 28 in.

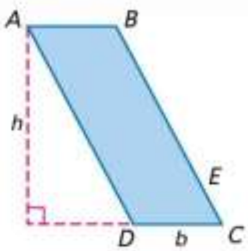
Area

The height given, DE, is 5 inches. $\overline{BC}$ is the base, which measures 10 inches.

$A = bh$ — Area of a parallelogram

$= (10)(5)$ or 50 in^2 — $b = 10$ and $h = 5$

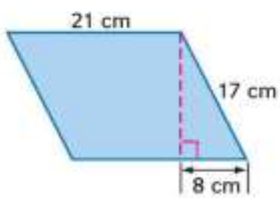
> **StudyTip**
>
> **Heights of Figures** The height of a figure can be measured by extending a base. In Example 1, the height of ▱*ABCD* that corresponds to base $\overline{DC}$ can be measured by extending $\overline{DC}$.



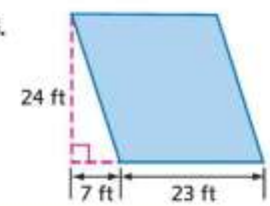
GuidedPractice

Find the perimeter and area of each parallelogram.

1A.



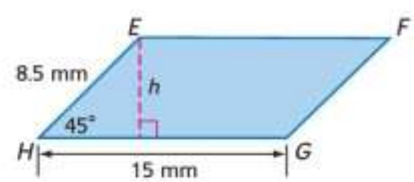
1B.



You may need to use trigonometry to find the area of a parallelogram.

Example 2 Area of a Parallelogram

Find the area of ▱*EFGH*.



Step 1 Use a 45°-45°-90° triangle to find the height h of the parallelogram.

Recall that if the measure of the leg opposite the 45° angle is h, then the measure of the hypotenuse is $h\sqrt{2}$.

$h\sqrt{2} = 8.5$ — Substitute 8.5 for the measure of the hypotenuse.

$h = \frac{8.5}{\sqrt{2}}$ or about 6 mm — Divide each side by $\sqrt{2}$.

Step 2 Find the area.

$A = bh$ — Area of a parallelogram

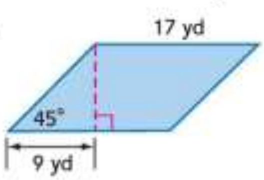
$\approx (15)(6)$ or 90 mm^2 — $b = 15$ and $h \approx 6$

> **WatchOut!**
>
> **Apply Math** Remember that perimeter is measured in linear units such as inches and centimeters. Area is measured in square units such as square feet and square millimeters.

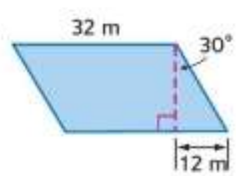
GuidedPractice

Find the area of each parallelogram. Round to the nearest tenth if necessary.

2A.



2B.

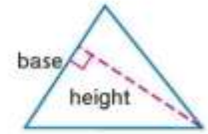


Review Vocabulary

altitude of a triangle
a segment from a vertex of a triangle to the line containing the opposite side and perpendicular to the line containing that side

2 Areas of Triangles

Like the base of a parallelogram, the **base of a triangle** can be any side. The **height of a triangle** is the length of an altitude drawn to a given base.



You can use the following postulate to develop the formula for the area of a triangle.

Postulate 11.2 Area Congruence Postulate

If two figures are congruent, then they have the same area.

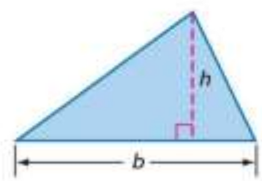
In the figures below, a parallelogram is cut in half along a diagonal to form two congruent triangles with the same base and height.

By the Area Congruence Postulate, the two congruent triangles have the same area. So, one triangle with base b and height h has half the area of a parallelogram with base b and height h.

Key Concept Area of a Triangle

Words The area A of a triangle is one half the product of a base b and its corresponding height h.

Symbols $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$



Real-World Link

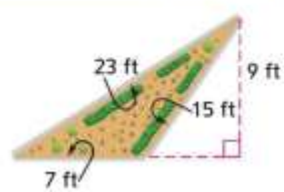
The Fort Worth Botanic Garden is the oldest botanic garden in Texas. It was established in 1934. The garden covers 109 acres with 2501 species of plants.

Source: Fort Worth Botanic Garden

Eileen Hart/E+/Getty Images

Real-World Example 3 Perimeter and Area of a Triangle

GARDENING D'Andre needs enough mulch to cover the triangular garden shown and enough paving stones to border it. If one bag of mulch covers 12 square feet and one paving stone provides a 4-inch border, how many bags of mulch and how many stones does he need to buy?



Step 1 Find the perimeter of the garden.

Perimeter of garden = 23 + 15 + 7 or 45 ft

Step 2 Find the area of the garden.

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(7)(9)$ or 31.5 ft^2 $b = 7$ and $h = 9$

Step 3 Use unit analysis to determine how many of each item are needed.

Bags of Mulch

$31.5 \text{ ft}^2 \cdot \frac{1 \text{ bag}}{12 \text{ ft}^2} = 2.625 \text{ bags}$

Paving Stones

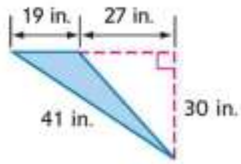
$45 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{1 \text{ stone}}{4 \text{ in.}} = 135 \text{ stones}$

Round the number of bags up so there is enough mulch. He will need 3 bags of mulch and 135 paving stones.

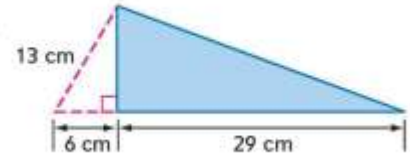
GuidedPractice

Find the perimeter and area of each triangle.

3A.



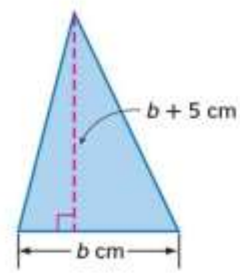
3B.



You can use algebra to solve for unknown measures in parallelograms and triangles.

Example 4 Use Area to Find Missing Measures

ALGEBRA The height of a triangle is 5 centimeters more than its base. The area of the triangle is 52 square centimeters. Find the base and height.



Step 1 Write expressions to represent each measure.

Let b represent the base of the triangle. Then the height is $b + 5$.

Step 2 Use the formula for the area of a triangle to find b.

$A = \frac{1}{2}bh$	Area of a triangle
$52 = \frac{1}{2}b(b + 5)$	Replace A with 52 and h with $b + 5$.
$104 = b(b + 5)$	Multiply each side by 2.
$104 = b^2 + 5b$	Distributive Property
$0 = b^2 + 5b - 104$	Subtract 104 from each side.
$0 = (b + 13)(b - 8)$	Factor.
$b + 13 = 0$ and $b - 8 = 0$	Zero Product Property
$b = -13$ $\quad$ $b = 8$	Solve for b.

StudyTip

Zero Product Property If the product of two factors is 0, then at least one of the factors must be 0.

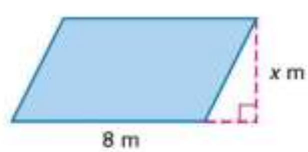
Step 3 Use the expressions from Step 1 to find each measure.

Since a length cannot be negative, the base measures 8 centimeters and the height measures 8 + 5 or 13 centimeters.

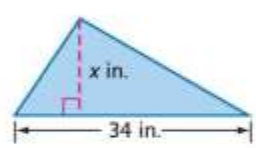
GuidedPractice

ALGEBRA Find x.

4A. $A = 148 \text{ m}^2$



4B. $A = 357 \text{ in}^2$



4C. **ALGEBRA** The base of a parallelogram is twice its height. If the area of the parallelogram is 72 square feet, find its base and height.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Examples 1–3 **Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.**

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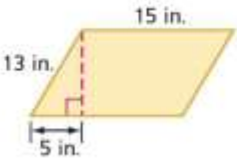

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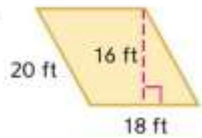

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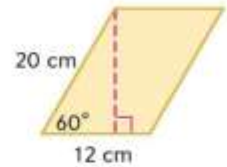

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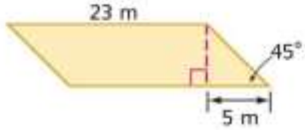

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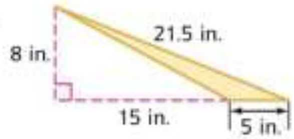

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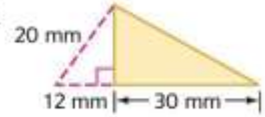
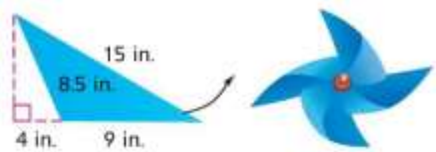
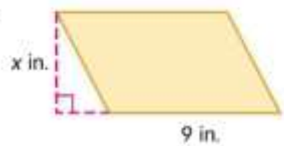
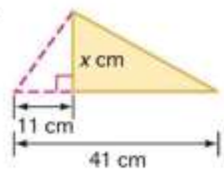


7. **CRAFTS** Marquez and Victoria are making pinwheels. Each pinwheel is composed of 4 triangles with the dimensions shown. Find the perimeter and area of one triangle.



Example 4 **Find *x*.**

8. $A = 153 \text{ in}^2$


9. $A = 165 \text{ cm}^2$


Practice and Problem Solving

Extra Practice is on page R11.

Examples 1–3 **MP ORGANIZE IDEAS** **Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.**

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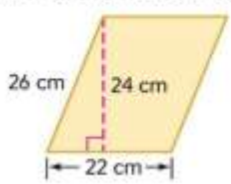

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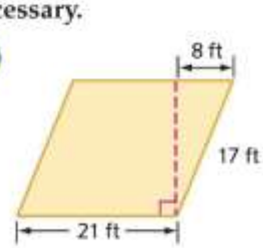

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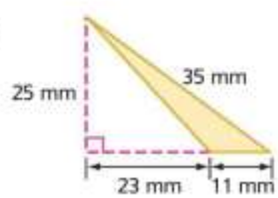

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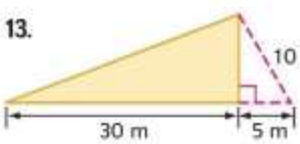

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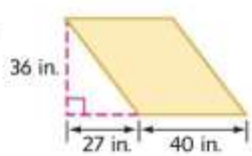

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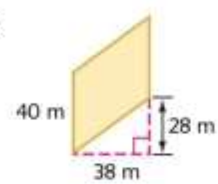
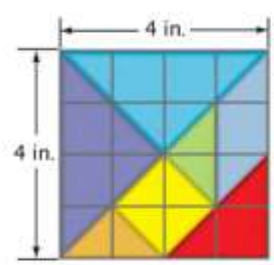


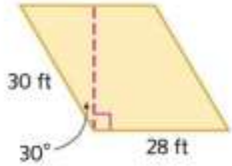
16. **TANGRAMS** The tangram shown is a 4-inch square.

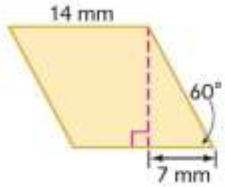
a. Find the perimeter and area of the purple triangle. Round to the nearest tenth.

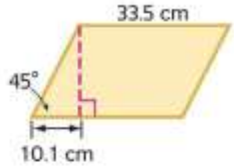
b. Find the perimeter and area of the blue parallelogram. Round to the nearest tenth.

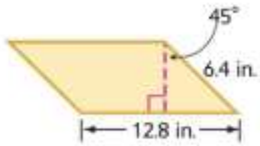


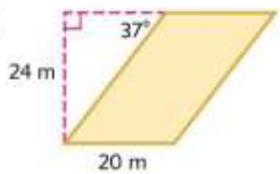
Example 2 **ORGANIZE IDEAS** **Find the area of each parallelogram. Round to the nearest tenth if necessary.**

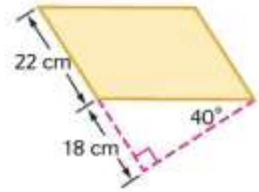
17. 

18. 

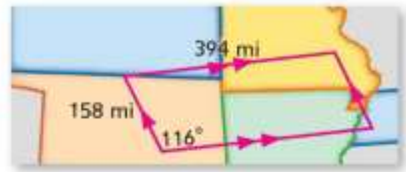
19. 

20. 

21. 

22. 

23. **WEATHER** Tornado watch areas are often shown on weather maps using parallelograms. What is the area of the region affected by the tornado watch shown? Round to the nearest square mile.



Example 4

24. The height of a parallelogram is 4 millimeters more than its base. If the area of the parallelogram is 221 square millimeters, find its base and height.

25. The height of a parallelogram is one fourth of its base. If the area of the parallelogram is 36 square centimeters, find its base and height.

26. The base of a triangle is twice its height. If the area of the triangle is 49 square feet, find its base and height.

27. The height of a triangle is 3 meters less than its base. If the area of the triangle is 44 square meters, find its base and height.

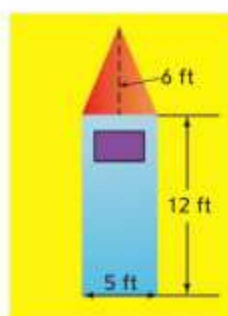
28. **FLAGS** Omar wants to make a replica of Guyana's national flag.

a. What is the area of the piece of fabric he will need for the red region? for the yellow region?

b. If the fabric costs $3.99 per square yard for each color and he buys exactly the amount of fabric he needs, how much will it cost to make the flag?

29. **MULTI-STEP** Madison is in charge of the set design for the drama club's rendition of *Romeo and Juliet*. The backdrop shown is 12 feet wide and 20 feet tall and needs 3 coats of paint. The window is 4 feet wide and 1 foot high. The paint store has the following available. One quart of paint covers 87.5 square feet.

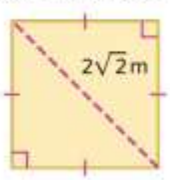
Size	8 oz	1 qt	1 gal
Cost ($)	3.75	14	30

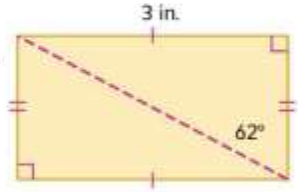


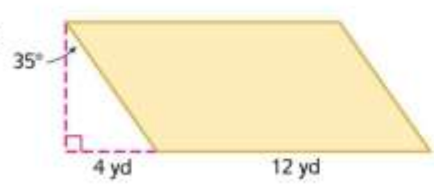
a. What should she buy to minimize cost?

b. Explain your solution process.

Find the perimeter and area of each figure. Round to the nearest hundredth, if necessary.

30. 

31. 

32. 

COORDINATE GEOMETRY **Find the area of each figure. Explain the method that you used.**

33. $\square ABCD$ with $A(4, 7)$, $B(2, 1)$, $C(8, 1)$, and $D(10, 7)$

34. $\triangle RST$ with $R(-8, -2)$, $S(-2, -2)$, and $T(-3, -7)$

35. **HERON'S FORMULA** Heron's Formula relates the lengths of the sides of a triangle to the area of the triangle. The formula is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is the *semiperimeter*, or one half the perimeter, of the triangle and a, b, and c are the side lengths.

a. Use Heron's Formula to find the area of a triangle with side lengths 7, 10, and 4.

b. Show that the areas found for a 5-12-13 right triangle are the same using Heron's Formula and using the triangle area formula you learned earlier in this lesson.

TEKS G.10(B)

36. MP **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between the area and perimeter of a rectangle.

a. Algebraic A rectangle has a perimeter of 12 units. If the length of the rectangle is x and the width of the rectangle is y, write equations for the perimeter and area of the rectangle.

b. Tabular Tabulate all possible whole-number values for the length and width of the rectangle, and find the area for each pair.

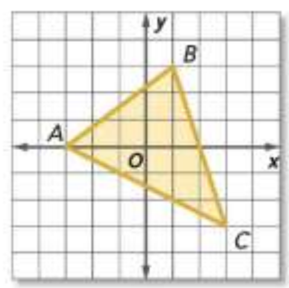
c. Graphical Graph the area of the rectangle with respect to its length.

d. Verbal Describe how the area of the rectangle changes as its length changes.

e. Analytical For what whole-number values of length and width will the area be greatest? least? Explain your reasoning.

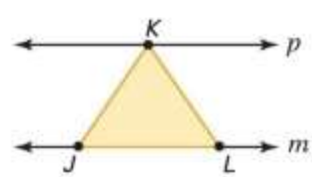
H.O.T. Problems Use Higher-Order Thinking Skills

37. MP **PROBLEM SOLVING** Find the area of $\triangle ABC$ graphed at the right. Explain your method.



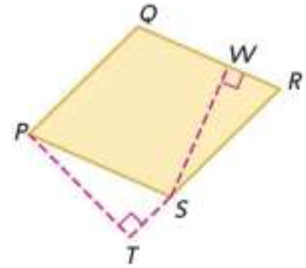
38. MP **JUSTIFY ARGUMENTS** Will the perimeter of a nonrectangular parallelogram *always*, *sometimes*, or *never* be greater than the perimeter of a rectangle with the same area and the same height? Explain.

39. **WRITING IN MATH** Points J and L lie on line m. Point K lies on line p. If lines m and p are parallel, describe how the area of $\triangle JKL$ will change as K moves along line p.



40. MP **ORGANIZE IDEAS** The area of a polygon is 35 square units. The height is 7 units. Draw three different triangles and three different parallelograms that meet these requirements. Label the base and height on each.

41. **WRITING IN MATH** Describe two different ways you could use measurement to find the area of parallelogram $PQRS$.

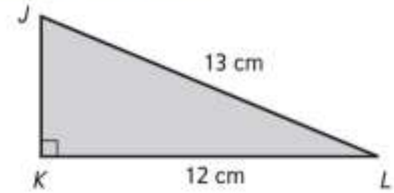


Preparing for Assessment

Example

TEKS G.10(B) MP G.1(B), G.1(F)

TEKS REVIEW Daevon draws the right triangle shown. Iris draws a right triangle in which each dimension is twice the corresponding dimension in Daevon's triangle. What is the area of Iris's triangle?



A 30 cm^2

B 60 cm^2

C 120 cm^2

D 312 cm^2

To find the area of a right triangle, you need to know the length of each leg. First find the length of $\overline{JK}$.

Apply the Pythagorean Theorem.

$JK^2 + 12^2 = 13^2$ Pythagorean Theorem

$JK^2 + 144 = 169$ Simplify.

$JK^2 = 25$ Subtract 144 from each side.

$JK = 5$ Take the square root of each side.

Iris's triangle has legs that are double the length of the corresponding leg of Daevon's triangle. So her triangle has legs of lengths 10 centimeters and 24 centimeters. Next, find the area of Iris's triangle.

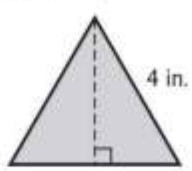
$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(24)(10)$ $b = 24, h = 10$

$= 120$ Multiply.

The area of Iris's triangle is 120 square centimeters. The correct answer is choice C.

42. **ACT/SAT** Katie makes coasters by cutting pieces of cardboard into equilateral triangles with the dimensions shown.



What is the area of each coaster?

A $2\sqrt{3}\text{ in}^2$

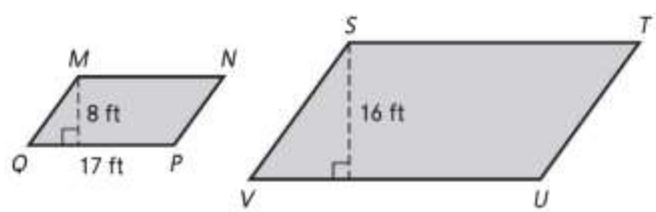
B 4 in^2

C $4\sqrt{2}\text{ in}^2$

D $4\sqrt{3}\text{ in}^2$

E $8\sqrt{3}\text{ in}^2$

43. The area of parallelogram *STUV* is 4 times the area of parallelogram *MNPQ*.



What is the length of $\overline{VU}$? TEKS G.10(B) MP G.1(C), G.1(F)

F 68 ft **H** 32 ft

G 34 ft **J** 25 ft

44. **GRIDDABLE** Ben uses software to draw a 45°-45°-90° triangle with a hypotenuse that is $3\sqrt{2}$ inches long. Then he uses the software to double all the dimensions of the triangle. What is the area of the new triangle, in square inches? TEKS G.10(B) MP G.1(C), G.1(F)

EXPLORE

11-2 Graphing Technology Lab
Areas of Trapezoids, Rhombi, and Kites

You can use the TI-Nspire Technology to explore special quadrilaterals.

Mathematical Processes

MP **G.1(C)** Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

Activity 1

Work cooperatively.

Step 1 Open a new **Graphs** page. Select **Show Grid** from the **View** menu so that points can be placed at integer coordinates.

Step 2 Select **Line** from the **Points & Lines** menu, and draw a horizontal line.

Step 3 Select **Parallel** from the **Construction** menu to draw a line parallel to your original line through a point with the same *x*-coordinate as a point in Step 2.

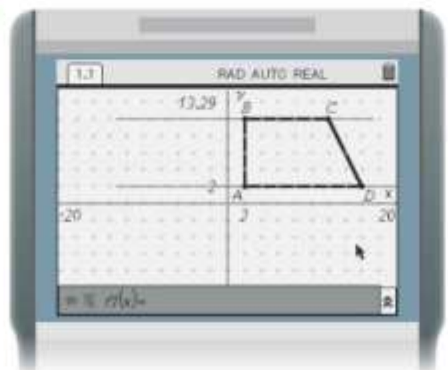
Step 4 Place an additional point on the parallel line you just constructed using **Point on** from the **Points & Lines** menu. Label the four points as shown.

Step 5 From the **Shapes** menu, select **Polygon**, and draw a polygon using the four points you created. From the **Actions** menu, select **Attributes**, select the polygon, and increase the line thickness of the polygon.

Step 6 Display the area of the polygon using the **Area** tool from the **Measurement** menu. Move each of the points and observe the effect on the area.

Step 1:

Step 5:



Step 6:

Analyze the Results **Work cooperatively.**

1. What type of quadrilateral is *ABCD*? Explain your reasoning.

2. **MAKE A CONJECTURE** Using the formulas you learned in Lesson 11-1, make a conjecture about the formula for the area of this type of quadrilateral if *BC* is b_1, *AD* is b_2, and *AB* is *h*. Explain.

(continued on the next page)

Areas of Trapezoids, Rhombi, and Kites *Continued*

Activity 2

Work cooperatively.

Step 1 Open a new **Graphs** page. Select **Show Grid** from the **View** menu so that points can be placed at integer coordinates.

Step 2 Select **Line** from the **Points & Lines** menu, and draw a line.

Step 3 Place a point above the line by selecting **Point** from the **Points & Lines** menu.

Step 4 Reflect the point above the line by choosing **Reflection** from the **Transformation** menu, then select the point and then the line.

Step 1:

Step 5 Label the four points as shown.

Step 6 From the **Shapes** menu, select **Polygon**, and draw a polygon using points W, X, Y, and Z.

Step 7 Display the area of the polygon using the **Area** tool from the **Measurement** menu. Move points W, X, and Y, and observe the effect on the area.

Step 6:

Step 8 Select **Segment** from the **Points & Lines** menu to draw the diagonals of $WXYZ$.

Step 9 Display the lengths of the diagonals using the **Length** tool from the **Measurement** menu, and display the angle between the diagonals using the **Angle** tool. Continue to move points W, X, and Y, and observe the effect on the area and the angle between the diagonals.

Step 8:

Analyze the Results **Work cooperatively.**

3. What type of quadrilateral is $WXYZ$? Explain your reasoning.

4. **MAKE A CONJECTURE** Using the formulas you learned in Lesson 11-1, develop a formula for the area of this type of quadrilateral. Let WY be d_1, and let XZ be d_2. Explain your reasoning.

5. **CHALLENGE** Construct a quadrilateral using two perpendicular lines and reflecting a point on each as you did in Step 4 of Activity 2. What type of quadrilateral is formed? Does the formula for the area you developed in Exercise 4 apply?

LESSON 11-2

Areas of Trapezoids, Rhombi, and Kites

Then

- You found areas of triangles and parallelograms.

Now

1. Find areas of trapezoids.
2. Find areas of rhombi and kites.

Why?

- Brianna has turned her hobby of making designer handbags and totes into a small business. Among her designs is a trapezoid-shaped handbag. To estimate the amount of material needed to produce each handbag, she needs to calculate the area of a trapezoid.

Mathematical Processes

G.1(E) Create and use representations to organize, record, and communicate mathematical ideas.

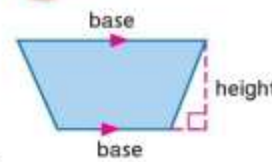
G.1(D) Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

NewVocabulary

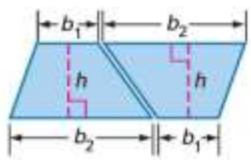
height of a trapezoid

1 Areas of Trapezoids

In Lesson 6-6, you learned that a *trapezoid* is a quadrilateral with exactly one pair of parallel sides. These parallel sides are called *bases*. The **height of a trapezoid** is the perpendicular distance between its bases.



In the figure below, a glide reflection of the first trapezoid results in two congruent trapezoids that fit together to form a parallelogram.

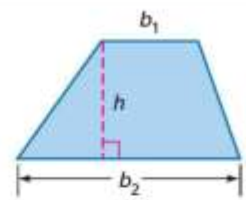


The area of the parallelogram is the product of the height h and the sum of the two bases, b_1 and b_2. The area of one trapezoid is one half the area of the parallelogram.

KeyConcept Area of a Trapezoid

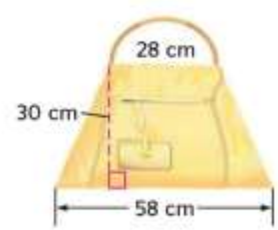
Words The area A of a trapezoid is one half the product of the height h and the sum of its bases, b_1 and b_2.

Symbols $A = \frac{1}{2}h(b_1 + b_2)$



Real-World Example 1 Area of a Trapezoid

CRAFTS One of Brianna's trapezoid-shaped totes is shown. Find the amount of material used to make the side shown.



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

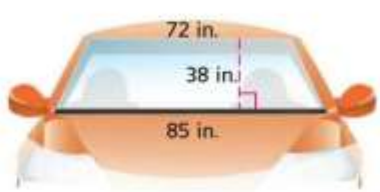
$= \frac{1}{2}(30)(28 + 58)$ $h = 30, b_1 = 28, b_2 = 58$

$= 1290$ Simplify.

The tote requires 1290 square centimeters.

GuidedPractice

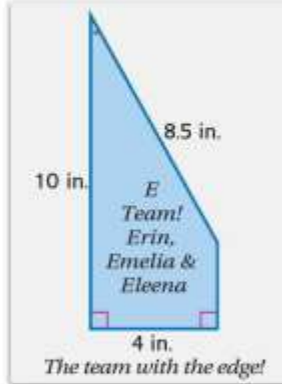
1. **AUTOMOBILES** Find the area of glass used to make the windshield of a van shown at the right.



Juice Images/age fotostock

Example 2 Area of a Trapezoid

JEWELRY Emelia designed the pennant shown for her team. Find the area of the shaded portion of her team's pennant.

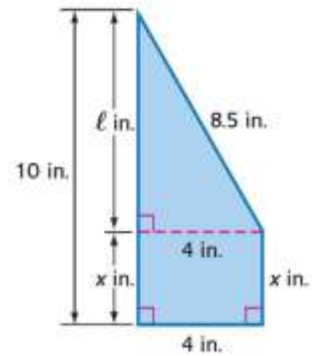


Read the Item

You are given a trapezoid with one base measuring 10 inches, a height of 4 inches, and a third side measuring 8.5 inches. To find the area of the trapezoid, first find the measure of the other base.

Solve the Item

Draw the segment shown to form a right triangle and a rectangle. The triangle has a hypotenuse of 8.5 inches and legs of 4 and ℓ inches. The rectangle has a length of 4 inches and a width of x inches.



Use the Pythagorean Theorem to find ℓ.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$\ell^2 + 4^2 = 8.5^2$	$a = \ell$, $b = 4$, and $c = 8.5$
$\ell^2 + 16 = 72.25$	Simplify.
$\ell^2 = 56.25$	Subtract 16 from each side.
$\ell = 7.5$	Take the positive square root of each side.

By Segment Addition, $\ell + x = 10$. So, $7.5 + x = 10$ and $x = 2.5$. The width of the rectangle is also the measure of the second base of the trapezoid.

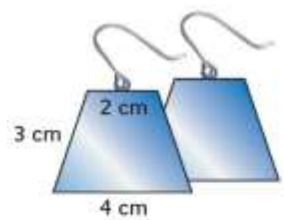
$A = \frac{1}{2}h(b_1 + b_2)$	Area of a trapezoid
$= \frac{1}{2}(4)(10 + 2.5)$	$h = 4$, $b_1 = 10$, and $b_2 = 2.5$
$= 25$	Simplify.

So the pennant has an area of 25 square inches.

CHECK The area of the trapezoid is the sum of the areas of the right triangle and rectangle. The area of the triangle is $\frac{1}{2}(4)(7.5)$ or 15 square inches. The area of the rectangle is $(4)(2.5)$ or 10 square inches. So the area of the trapezoid is $15 + 10$ or 25 square inches. ✓

GuidedPractice

2. **JEWELRY** Owen designed the silver earrings shown that are shaped like isosceles trapezoids. What is the area of each earring?



StudyTip

Separating Figures To solve some area problems, you need to draw in parallel and/or perpendicular lines to find information not provided.

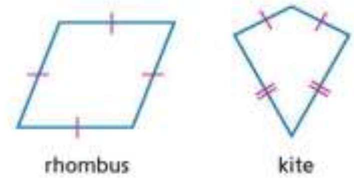
Real-WorldCareer

Craft Artist Craft artists create their art by hand to sell or exhibit. They work with a wide variety of materials including textiles, woods, metal, and ceramics. Most artists receive some type of postsecondary training, and about 63% are self-employed. Craft artists make up about 3% of all artists.

Richard Hutchings/Digital Light Source

2 Areas of Rhombi and Kites

Recall from Lessons 6-5 and 6-6 that a *rhombus* is a parallelogram with all four sides congruent and a *kite* is a quadrilateral with exactly two pairs of consecutive congruent sides.



ReviewVocabulary

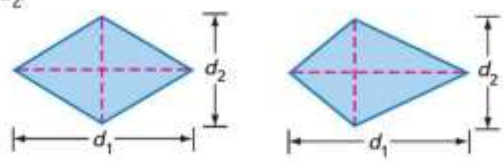
diagonal a segment that connects any two nonconsecutive vertices in a polygon

The areas of rhombi and kites are related to the lengths of their diagonals.

KeyConcept Area of a Rhombus or Kite

Words The area A of a rhombus or kite is one half the product of the lengths of its diagonals, d_1 and d_2.

Symbols $A = \frac{1}{2}d_1d_2$

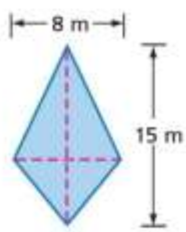


You will derive the formulas for the area of a kite and the area of a rhombus in Exercises 23 and 24.

Example 3 Area of a Rhombus and a Kite

Find the area of each rhombus or kite.

a.

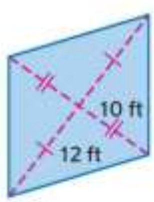


$A = \frac{1}{2}d_1d_2$ Area of a kite

$= \frac{1}{2}(8)(15)$ $d_1 = 8$ and $d_2 = 15$

$= 60 \text{ m}^2$ Simplify.

b.



Step 1 Find the length of each diagonal.

Since the diagonals of a rhombus bisect each other, then lengths of the diagonals are 12 + 12 or 24 feet and 10 + 10 or 20 feet.

Step 2 Find the area of the rhombus.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

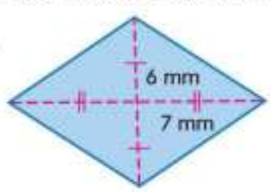
$= \frac{1}{2}(24)(20)$ $d_1 = 24$ and $d_2 = 20$

$= 240 \text{ ft}^2$ Simplify.

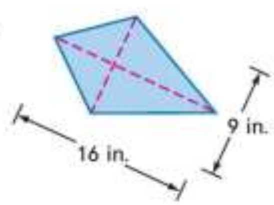
GuidedPractice

Find the area of each rhombus or kite.

3A.



3B.



Apic/Hulton Archive/Getty Images

Math HistoryLink

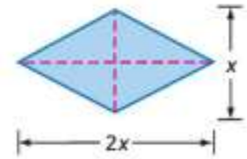
Heron of Alexandria

(c. 10–70 A.D.) Heron was a mathematician and engineer in Roman Egypt. He developed a formula for finding the area of a triangle if the lengths of the sides are known.

You can use algebra to solve for unknown measures in trapezoids, rhombi, and kites.

Example 4 Use Area to Find Missing Measures

ALGEBRA One diagonal of a rhombus is twice as long as the other diagonal. If the area of the rhombus is 169 square millimeters, what are the lengths of the diagonals?



Step 1 Write an expression to represent each measure.

Let x represent the length of one diagonal. Then the length of the other diagonal is $2x$.

Step 2 Use the formula for the area of a rhombus to find x.

$A = \frac{1}{2}d_1d_2$	Area of a rhombus
$169 = \frac{1}{2}(x)(2x)$	$A = 169$, $d_1 = x$, and $d_2 = 2x$
$169 = x^2$	Simplify.
$13 = x$	Take the positive square root of each side.

So the lengths of the diagonals are 13 millimeters and 2(13) or 26 millimeters.

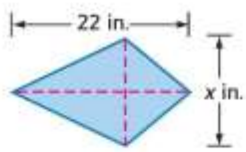
Go Online!

The area formulas in this Concept Summary are important to remember. Log into your **eStudent Edition** to bookmark this page.

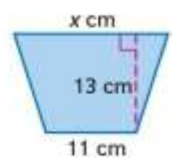
GuidedPractice

ALGEBRA Find x.

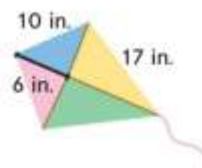
4A. $A = 92 \text{ in}^2$



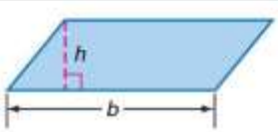
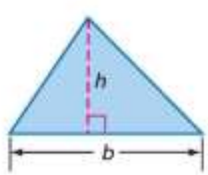
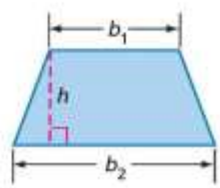
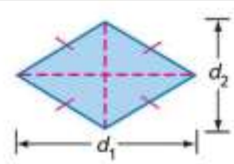
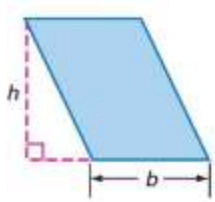
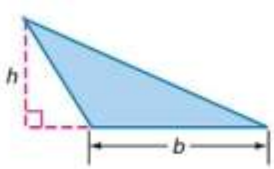
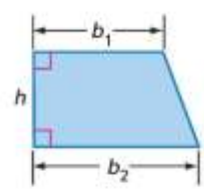
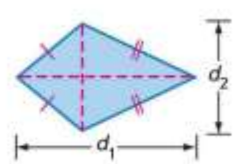
4B. $A = 177 \text{ cm}^2$



4C. ALGEBRA What is the area of the kite shown?



ConceptSummary Areas of Polygons

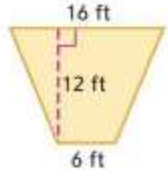
Parallelogram	Triangles	Trapezoids	Rhombi and Kites
			
			
$A = bh$	$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(b_1 + b_2)$	$A = \frac{1}{2}d_1d_2$

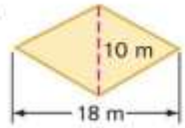
Check Your Understanding

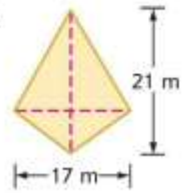
● = Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

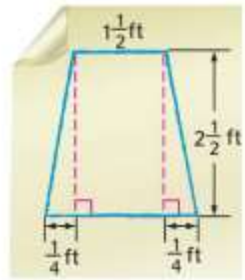
Examples 1–3 **Find the area of each trapezoid, rhombus, or kite.**

1. 

2. 

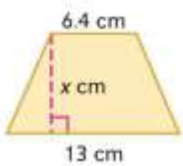
3. 

4. **PEP RALLY** Suki is designing posters for the Homecoming game. Her design is shown at the right. What is the area of the poster in square feet?

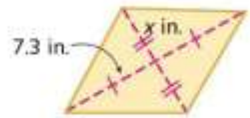


Example 4 **ALGEBRA** **Find x.**

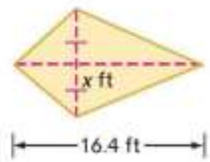
5. $A = 78 \text{ cm}^2$



6. $A = 96 \text{ in}^2$



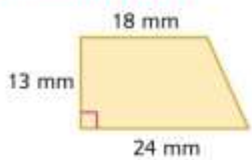
7. $A = 104 \text{ ft}^2$

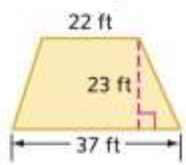


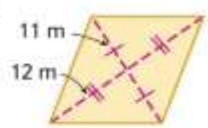
Practice and Problem Solving

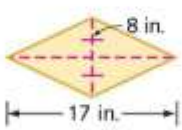
Extra Practice is on page R11.

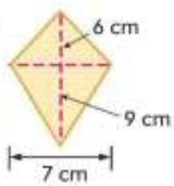
Examples 1–3 **ORGANIZE IDEAS** **Find the area of each trapezoid, rhombus, or kite.**

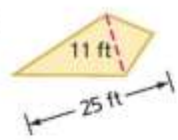
8. 

9. 

10. 

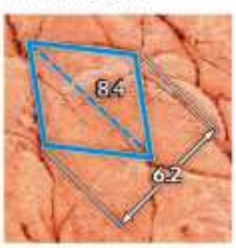
11. 

12. 

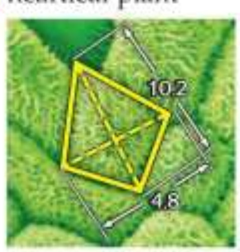
13. 

MICROSCOPES **Find the area of the identified portion of each magnified image. Assume that the identified portion is either a trapezoid, rhombus, or kite. Measures are provided in microns.**

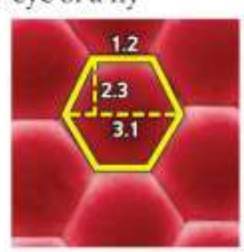
14. human skin



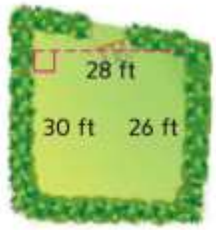
15. heartleaf plant



16. eye of a fly



17. **JOBS** Jimmy works on his neighbors' yards after school to earn extra money to buy a car. He is going to plant grass seed in Mr. Troyer's yard. What is the area of the yard?



Example 4 **ALGEBRA Find each missing length.**

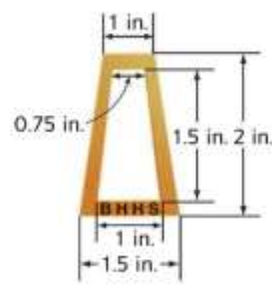
18. One diagonal of a kite is twice as long as the other diagonal. If the area of the kite is 240 square inches, what are the lengths of the diagonals?

19. The area of a rhombus is 168 square centimeters. If one diagonal is three times as long as the other, what are the lengths of the diagonals?

20. A trapezoid has base lengths of 12 and 14 feet with an area of 322 square feet. What is the height of the trapezoid?

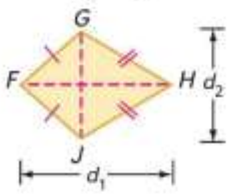
21. A trapezoid has a height of 8 meters, a base length of 12 meters, and an area of 64 square meters. What is the length of the other base?

22. **HONORS** Estella has been asked to join an honor society at school. Before the first meeting, new members are asked to sand and stain the front side of a piece of wood in the shape of an isosceles trapezoid. What is the surface area that Estella will need to sand and stain?

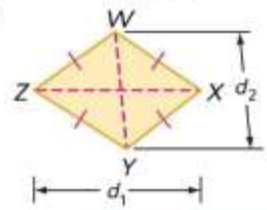


For each figure, provide a justification showing that $A = \frac{1}{2}d_1d_2$.

23.



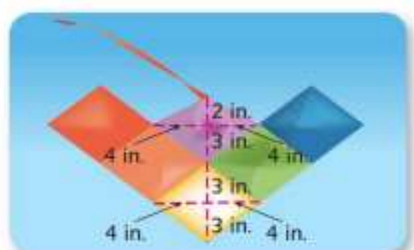
24.



25. **CRAFTS** Ashanti is competing in the Zilker Kite Festival. The yellow, red, orange, green, and blue pieces of her kite design shown are congruent rhombi.

   a. How much fabric of each color does she need to buy?

   b. Ashanti wants the total area of her kite to be no greater than 200 square inches. Does her kite meet this requirement? Explain.



ORGANIZE IDEAS Find the area of each quadrilateral with the given vertices.

26. $A(-8, 6)$, $B(-5, 8)$, $C(-2, 6)$, and $D(-5, 0)$

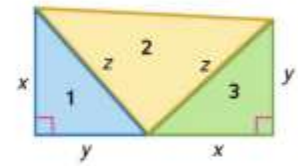
27. $W(3, 0)$, $X(0, 3)$, $Y(-3, 0)$, and $Z(0, -3)$

28. **METALS** When magnified in very powerful microscopes, some metals are composed of grains that have various polygonal shapes.

   a. What is the area of figure 1 if the grain has a height of 4 microns and bases with lengths of 5 and 6 microns?

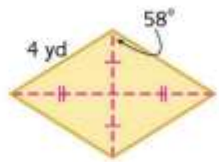
   b. If figure 2 has perpendicular diagonal lengths of 3.8 microns and 4.9 microns, what is the area of the grain?

29. **PROOF** The figure at the right is a trapezoid that consists of two congruent right triangles and an isosceles triangle. In 1876, James A. Garfield, the 20th president of the United States, discovered a proof of the Pythagorean Theorem using this diagram. Prove that $x^2 + y^2 = z^2$.

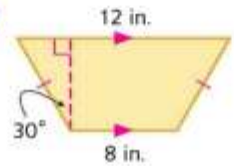


DIMENSIONAL ANALYSIS Find the perimeter and area of each figure in feet. Round to the nearest tenth, if necessary.

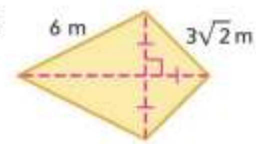
30.



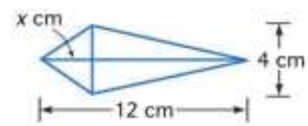
31.



32.



33. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate perimeters of kites.



a. Geometric Draw a kite like the one shown if $x = 2$.

b. Geometric Repeat the process in part **a** for three x-values between 2 and 10 and for an x-value of 10. The overall length of the kite should remain 12 centimeters.

c. Tabular Measure and record in a table the perimeter of each kite, along with the x-value.

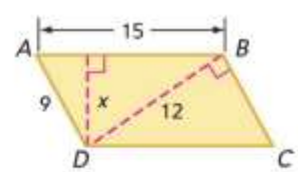
d. Graphical Graph the perimeter versus the x-value using the data from your table.

e. Analytical Make a conjecture about the value of x that will minimize the perimeter of the kite. What is the significance of this value?

H.O.T. Problems Use Higher-Order Thinking Skills

34. **ERROR ANALYSIS** Antonio and Madeline want to draw a trapezoid that has a height of 4 units and an area of 18 square units. Antonio says that only one trapezoid will meet the criteria. Madeline disagrees and thinks that she can draw several different trapezoids with a height of 4 units and an area of 18 square units. Is either of them correct? Explain your reasoning.

35. **PROBLEM SOLVING** Find x in parallelogram $ABCD$.



36. **ORGANIZE IDEAS** Draw a kite and a rhombus with an area of 6 square inches. Label and justify your drawings.

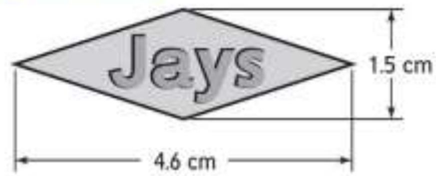
37. **ANALYZE RELATIONSHIPS** If the areas of two rhombi are equal, are the perimeters *sometimes*, *always*, or *never* equal? Explain.

38. **WRITING IN MATH** How can you use trigonometry to find the area of a figure?

Example

TEKS G.10(B) MP G.1(A), G.1(B)

TEKS REVIEW The Jays' team logo is a rhombus, as shown. The team's manager makes an enlargement of the logo in which each dimension is 4 times larger than the version shown here. Which of the following best describes how the areas of the logos compare?



A The area of the enlargement is 2 times the area of the original.

B The area of the enlargement is 4 times the area of the original.

C The area of the enlargement is 8 times the area of the original.

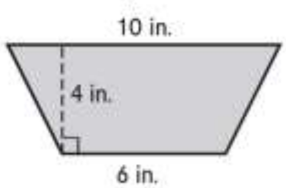
D The area of the enlargement is 16 times the area of the original.

You are given the lengths of the diagonals of the original rhombus. The lengths of the diagonals of the enlarged rhombus are $4 \cdot 1.5 = 6$ centimeters and $4 \cdot 4.6 = 18.4$ centimeters. Next find the areas of the original and enlarged rhombi.

Original rhombus		Enlarged rhombus
$A = \frac{1}{2}d_1d_2$	Area of a rhombus	$A = \frac{1}{2}d_1d_2$
$= \frac{1}{2}(1.5)(4.6)$	Substitution	$= \frac{1}{2}(6)(18.4)$
$= 3.45 \text{ cm}^2$	Multiply.	$= 55.2 \text{ cm}^2$

Compare the areas: $55.2 \div 3.45 = 16$. The area of the rhombus in the enlarged logo is 16 times greater than the area of the original rhombus. The correct answer is choice D.

39. **ACT/SAT** Li Mei drew the trapezoid shown below. Tom drew a trapezoid with the same bases, but with a height that is 2 inches greater than the height of Li Mei's trapezoid.



Which of the following statements about the areas of the trapezoids is true?

A The area of Tom's trapezoid is 2 square inches greater than the area of Li Mei's trapezoid.

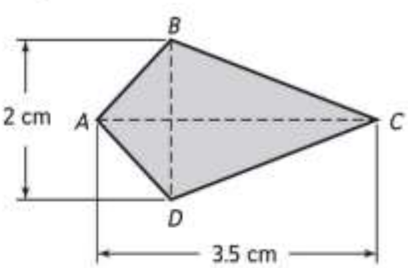
B The area of Tom's trapezoid is 4 times the area of Li Mei's trapezoid.

C The area of Tom's trapezoid is 16 square inches greater than the area of Li Mei's trapezoid.

D The area of Tom's trapezoid is 28 square inches greater than the area of Li Mei's trapezoid.

E The area of Tom's trapezoid is 32 square inches greater than the area of Li Mei's trapezoid.

40. The figure shows a kite that Alex created using geometry software.



Which of the following will result in a kite with twice the area of $ABCD$? TEKS G.10(B) MP G.1(F), G.1(G)

I. Double the length of each diagonal.

II. Double the length of $\overline{BD}$.

III. Double the length of $\overline{AC}$.

F I only

G II only

H III only

J II and III only

EXTEND

11-2 Geometry Lab
Population Density

After data are collected for the U.S. census, the population density is calculated for states, major cities, and other areas. **Population density** is the measurement of population per unit of area.

Mathematical Processes

MP **G.1(A)** Apply mathematics to problems arising in everyday life, society, and the workplace. *Also addresses G.1(C).*

Activity 1 Calculate Population Density

Find the population density for Tarrant County using the data in the table.

Calculate population density with the formula population density $= \frac{\text{population}}{\text{land area}}$.

The population density of Tarrant County would be $\frac{1{,}809{,}034}{897}$ or about 2017 people per square mile.

Texas County	Population	Land Area (mi^2)
Zavala	11,677	1302
Bexar	1,714,773	1257
Tarrant	1,809,034	897
Matagorda	36,702	1612
Hudspeth	3476	4572

Model and Analyze Work cooperatively.

1. Find the population densities for Zavala, Bexar, Matagorda, and Hudspeth counties. Round to the nearest person. Of the five counties, which have the highest and lowest population densities?

Activity 2 Use Population Density

In a proposal to establish a new rustic campground at Yellowstone National Park, there is a concern about the number of wolves in the area. At last report, there were 98 wolves in the park. The new campground will be accepted if there are fewer than 2 wolves in the campground. Use the data in the table to determine if the new campground can be established.

Location	Size
Area of park	3472 mi^2
Area of new campground	10 acres

Step 1 Find the density of wolves in the park.
$98 \div 3472 = 0.028$ wolves per square mile

Step 2 Find the density of wolves in the proposed campground. First convert the size of the campground to square miles. If 1 acre is equivalent to 0.0015625 square mile, then 10 acres is 0.015625 square mile. The potential number of wolves in the proposed site is $0.015625 \cdot 0.028$ or 0.0004375 wolves.

Step 3 Since 0.0004375 is fewer than 2, the proposed campground can be accepted.

Exercises

2. Find the population density of gaming system owners in the United States if there are 64,288,000 owners and the area of the United States is 3,794,083 square miles.

3. The population density of the burrowing owl in Cape Coral, Florida, is 8.3 pairs per square mile. A new golf club is planned for a 2.4-square-mile site where the owl population is estimated to be 17 pairs. Would Lee County approve the proposed club if their policy is to decline when the estimated population density of owls is below the average density? Explain.

LESSON 11-3 Areas of Circles and Sectors

Then

- You found the circumference of a circle.

Now

1. Find areas of circles.
2. Find areas of sectors of circles.

Why?

- Because a pizza place in San Antonio has a 42" pizza, they were featured on a TV show. To determine whether a medium, large, or extra-large pizza is a better value, you can compare the cost per square inch. Divide the cost of each pizza by its area.

Targeted TEKS

G.11(B) Determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure.

G.12(C) Apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems.

Mathematical Processes

G.1(A) Apply mathematics to problems arising in everyday life, society, and the workplace.

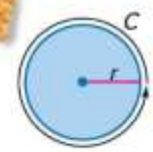
G.1(E) Create and use representations to organize, record, and communicate mathematical ideas.

NewVocabulary

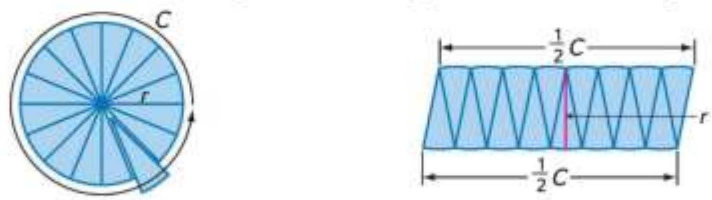
sector of a circle
segment of a circle

1 Areas of Circles

In Lesson 10-1, you learned that the formula for the circumference C of a circle with radius r is given by $C = 2\pi r$. You can use this formula to develop the formula for the area of a circle.



Below, a circle with radius r and circumference C has been divided into congruent pieces and then rearranged to form a figure that resembles a parallelogram.

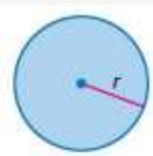


As the number of congruent pieces increases, the rearranged figure more closely approaches a parallelogram. The base of the parallelogram is $\frac{1}{2}C$ and the height is r, so its area is $\frac{1}{2}C \cdot r$. Since $C = 2\pi r$, the area of the parallelogram is also $\frac{1}{2}(2\pi r)r$ or πr^2.

KeyConcept Area of a Circle

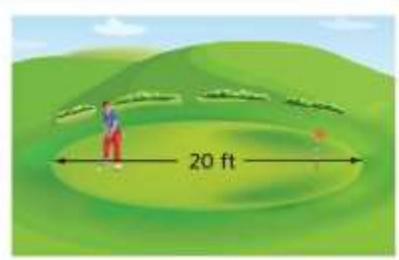
Words The area A of a circle is equal to π times the square of the radius r.

Symbols $A = \pi r^2$



Real-World Example 1 Area of a Circle

SPORTS What is the area of the circular putting green shown to the nearest square foot?



The diameter is 20 feet, so the radius is 10 feet.

$A = \pi r^2$ Area of a circle

$= \pi(10)^2$ $r = 10$

≈ 314 Use a calculator.

So, the area is about 314 square feet.

GuidedPractice

1. **SPORTS** An archery target has a radius of 12 inches. What is the area of the target to the nearest square inch?

Corbis Premium RF/Alamy

Example 2 Use the Area of a Circle to Find a Missing Measure

ALGEBRA Find the radius of a circle with an area of 95 square centimeters.

$A = \pi r^2$	Area of a circle
$95 = \pi r^2$	$A = 95$
$\frac{95}{\pi} = r^2$	Divide each side by π.
$5.5 \approx r$	Use a calculator. Take the positive square root of each side.

The radius of the circle is about 5.5 centimeters.

GuidedPractice

2. **ALGEBRA** The area of a circle is 196π square yards. Find the diameter.

ReviewVocabulary

central angle an angle with a vertex in the center of a circle and with sides that contain two radii of the circle

arc a portion of a circle defined by two endpoints

2 Areas of Sectors

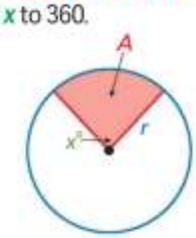
A slice of a circular pizza is an example of a sector of a circle. A **sector of a circle** is a region of a circle bounded by a central angle and its intercepted major or minor arc. The formula for the area of a sector is similar to the formula for arc length.

KeyConcept Area of a Sector

The ratio of the **area A of a sector** to the **area of the whole circle, πr^2**, is equal to the ratio of the **degree measure of the intercepted arc x** to 360.

Proportion: $\frac{A}{\pi r^2} = \frac{x}{360}$

Equation: $A = \frac{x}{360} \cdot \pi r^2$



Picturenet/Blend Images/Getty Images

Real-WorldLink

About 3 billion pizzas are sold each year in the United States. That is equivalent to about 46 slices per person annually.

Source: Statistic Brain

TEKS G.12(C)

Real-World Example 3 Area of a Sector

PIZZA A circular pizza has a diameter of 12 inches and is cut into 8 congruent slices. What is the area of one slice to the nearest hundredth?

Step 1 Find the arc measure of a pizza slice.

Since the pizza is equally divided into 8 slices, each slice will have an arc measure of $360 \div 8$ or 45.

Step 2 Find the radius of the pizza. Use this measure to find the area of the sector, or slice.

The diameter is 12 inches, so the radius is 6 inches.

$A = \frac{x}{360} \cdot \pi r^2$	Area of a sector
$= \frac{45}{360} \cdot \pi(6)^2$	$x = 45$ and $r = 6$
≈ 14.14	Use a calculator.

So, the area of one slice of this pizza is about 14.14 square inches.

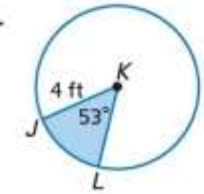
Go Online!

Watch **Personal Tutor** videos to hear descriptions of solving problems involving areas of circles and sectors. Try to describe how to solve a problem for a partner. Have them ask you questions to help your understanding.

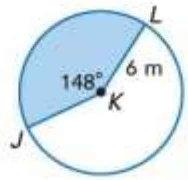
Guided Practice

Find the area of the shaded sector. Round to the nearest tenth.

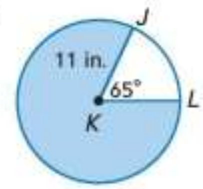
3A.



3B.



3C.



3D. **CRAFTS** The color wheel at the right is a tool that artists use to organize color schemes. If the diameter of the wheel is 10 inches and each of the 12 sections is congruent, find the approximate area covered by green hues.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Example 1 **CONSTRUCTION** **Find the area of each circle. Round to the nearest tenth.**

1.

2.

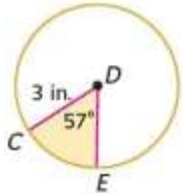
Example 2 **Find the indicated measure. Round to the nearest tenth.**

3. Find the diameter of a circle with an area of 74 square millimeters.

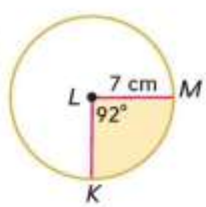
4. The area of a circle is 88 square inches. Find the radius.

Example 3 TEKS G.12(C) **Find the area of each shaded sector. Round to the nearest tenth.**

5.



6.



7. **BAKING** Chelsea is baking pies for a fundraiser at her school. She divides each 9-inch pie into 6 equal slices.

a. What is the area, in square inches, for each slice of pie?

b. If each slice costs $0.25 to make and she sells 8 pies at $1.25 for each slice, how much money will she raise?

Example 1

APPLY MATH **Find the area of each circle. Round to the nearest tenth.**

8.

9.

10.

11.

12.

13.

Example 2

Find the indicated measure. Round to the nearest tenth, if necessary.

14. The area of a circle is 68 square centimeters. Find the diameter.

15. Find the diameter of a circle with an area of 94 square millimeters.

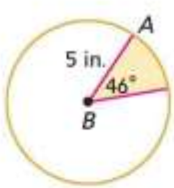
16. The area of a circle is 112 square inches. Find the radius.

17. Find the radius of a circle with an area of 206 square feet.

Example 3
TEKS G.12(C)

Find the area of each shaded sector. Round to the nearest tenth, if necessary.

18.



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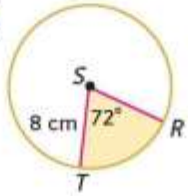

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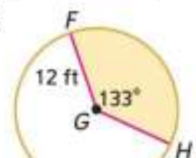

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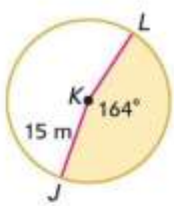

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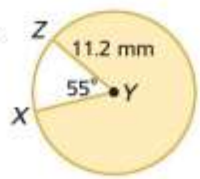

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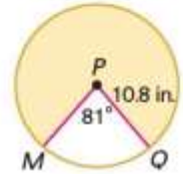
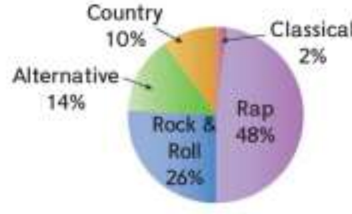


24. MUSIC The music preferences of students at Thomas Jefferson High are shown in the circle graph. Find the area of each sector and the degree measure of each intercepted arc if the radius of the circle is 1 unit.



25. JEWELRY A jeweler makes a pair of earrings by cutting two 50° sectors from a silver disk.

a. Find the area of each sector.

b. If the weight of the silver disk is 2.3 grams, how many milligrams does the silver wedge for each earring weigh?

26. PROM Students voted on their favorite prom theme.

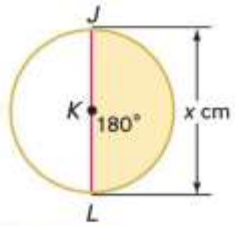
a. Create a circle graph with a diameter of 2 inches to represent these data.

b. Find the area of each theme's sector in your graph. Round to the nearest hundredth of an inch.

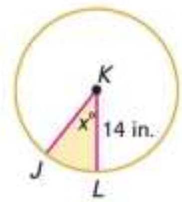
Theme	Percent
An Evening of Stars	11
Mardi Gras	32
Springtime in Paris	8
Night in Times Square	47
Undecided	2

ORGANIZE IDEAS **The area A of each shaded region is given. Find x.**

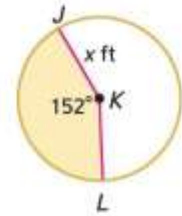
27. $A = 66 \text{ cm}^2$



28. $A = 94 \text{ in}^2$



29. $A = 128 \text{ ft}^2$



30. MULTI-STEP Luna is organizing a banquet for the Honor Society, and she needs 13 tablecloths for the round tables in the hall. The area of each table is approximately 29.27 square feet. She can rent tablecloths for $16 each or she can make them herself. Her local fabric store carries three different bolts of suitable fabric. The standard bolt is 60 inches wide and 100 yards long and costs $75. The wide bolt is 81 inches wide, 25 yards long, and costs $125. The extra-wide bolt is 90 inches wide, 25 yards long, and costs $150. Each tablecloth should cover the table with 9 inches of overhang.

a. How can Luna minimize the cost of the tablecloths?

b. Explain your reasoning.

c. What assumptions did you make?

31. TREES The age of a living tree can be determined by multiplying the diameter of the tree by its growth factor, or rate of growth. The Coast Live Oak is the largest tree in Texas.

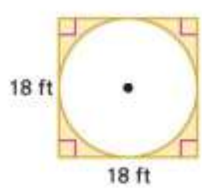
a. What is the diameter of a live oak tree with a circumference of 36 feet?

b. If the growth factor of the live oak tree is 130, what is the age of the tree?

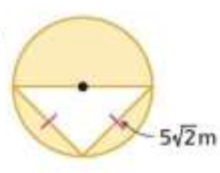
Find the area of the shaded region. Round to the nearest tenth.

TEKS G.11(B)

32.



33.



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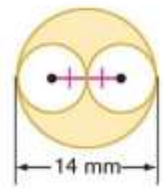

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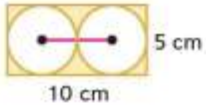

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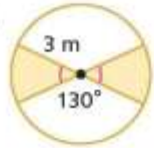
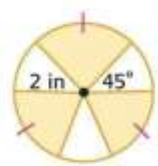
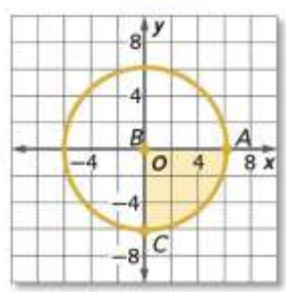


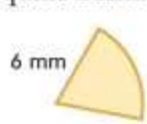
37.



38. COORDINATE GEOMETRY What is the area of sector ABC shown on the graph?

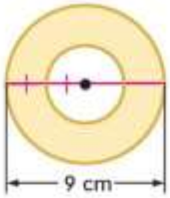


39. ALGEBRA The figure shown below is a sector of a circle. If the perimeter of the figure is 22 millimeters, find its area in square millimeters.



Find the area of each shaded region.

40.



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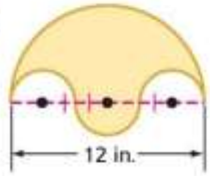
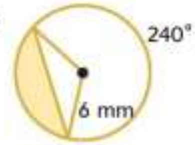


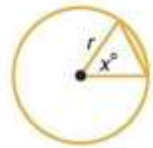
42.



Example 4

43. MP **MULTIPLE REPRESENTATIONS** In this problem, you will investigate segments of circles. A **segment of a circle** is the region bounded by an arc and a chord.

a. Algebraic Write an equation for the area A of a segment of a circle with a radius r and a central angle of $x°$. (*Hint*: Use trigonometry to find the base and height of the triangle.)



b. Tabular Calculate and record in a table ten values of A for x-values ranging from 10 to 90 if r is 12 inches. Round to the nearest tenth.

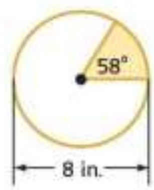
c. Graphical Graph the data from your table with the x-values on the horizontal axis and the A-values on the vertical axis.

d. Analytical Use your graph to predict the value of A when x is 63. Then use the formula you generated in part **a** to calculate the value of A when x is 63. How do the values compare?

TEKS G.12(C)

H.O.T. Problems Use Higher-Order Thinking Skills

44. ERROR ANALYSIS Kristen and Chase want to find the area of the shaded region in the circle shown. Is either of them correct? Explain your reasoning.



Kristen

$A = \frac{x}{360} \cdot \pi r^2$

$= \frac{58}{360} \cdot \pi(8)^2$

$= 32.4 \text{ in}^2$

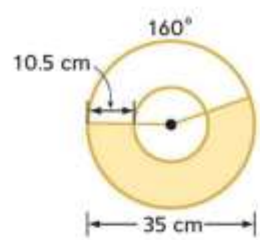
Chase

$A = \frac{x}{360} \cdot \pi r^2$

$= \frac{58}{360} \cdot \pi(4)^2$

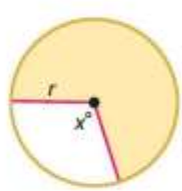
$= 8.1 \text{ in}^2$

45. MP **PROBLEM SOLVING** Find the area of the shaded region. Round to the nearest tenth.



46. MP **JUSTIFY ARGUMENTS** Refer to Exercise 43. Is the area of a sector of a circle *sometimes*, *always*, or *never* greater than the area of its corresponding segment?

47. WRITING IN MATH Describe two methods you could use to find the area of the shaded region of the circle. Which method do you think is more efficient? Explain your reasoning.



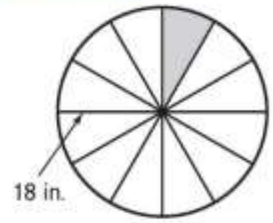
48. MP **APPLY MATH** Derive the formula for the area of a sector of a circle using the formula for arc length.

49. WRITING IN MATH If the radius of a circle doubles, will the measure of a sector of that circle double? Will it double if the arc measure of that sector doubles?

Example

TEKS G.12(C) MP G.1(A), G.1(C)

TEKS REVIEW Visitors at a school carnival have a chance to toss a bean onto a circular tabletop that is divided into equal sectors, as shown. Visitors win a prize if the bean lands in the red sector. What is the area of this sector, in square inches? Round to the nearest tenth.



You are given the radius of the circle. In order to find the area of the sector, you need to know the measure of the intercepted arc.

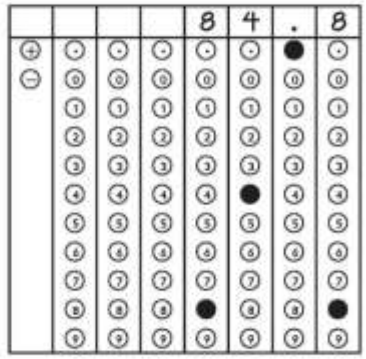
Since the tabletop is equally divided into 12 sectors, each sector has an arc that measures $360° \div 12 = 30°$.

$A = \frac{x}{360} \cdot \pi r^2$ Area of a sector

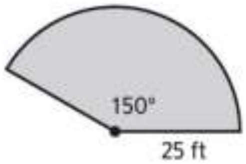
$= \frac{30}{360} \cdot \pi(18)^2$ $x = 30$ and $r = 18$

≈ 84.8 Use a calculator.

Write 84.8 at the top of the grid and fill in the appropriate bubbles.



50. A lawn sprinkler sprays water 25 feet and moves back and forth through an angle of 150°.



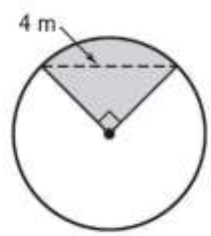
Which of the following is the best estimate of the area of the lawn that gets watered? TEKS G.12(C) MP G.1(A), G.1(C)

A 65 ft^2
B 818 ft^2
C 1963 ft^2
D 4712 ft^2

51. **GRIDDABLE** A sector of a circle has an intercepted arc that measures 120°. The area of the sector is 155.8 square centimeters. What is the radius of the circle in centimeters? Round to the nearest tenth. TEKS G.12(C) MP G.1(C), G.1(F)

52. **GRIDDABLE** A regular hexagon is divided into 6 congruent triangles. If the perimeter of the hexagon is 48 centimeters, what is the height of each triangle to the nearest hundredth? TEKS G.11(A) MP G.1(E)

53. **ACT/SAT** Which expression represents the area of the shaded sector in square meters?



F $\frac{\sqrt{2}}{2}\pi$
G $\sqrt{2}\pi$
H 2π
J 4π
K 8π

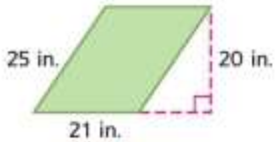
54. In $\odot C$, a sector has an area of 24π square inches. The radius of $\odot C$ is 12 inches. What is the measure, in degrees, of the arc that is intercepted by the sector? TEKS G.12(C) MP G.1(B), G.1(F)

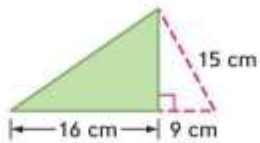
A 360
B 60π
C 60
D $\frac{180}{\pi}$

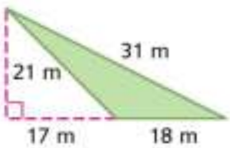
Mid-Chapter Quiz

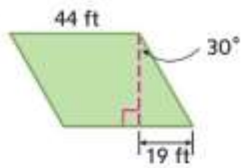
Lessons 11-1 through 11-3

Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary. (Lesson 11-1)

1. 

2. 

3. 

4. 

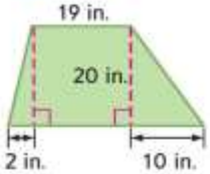
5. The height of a triangle is 8 inches more than its base. The area of the triangle is 104.5 square inches. Find the base and height. (Lesson 11-1)

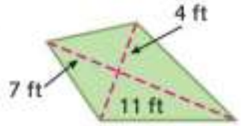
6. **DESIGN** A plaque is made with a rhombus in the middle. If the diagonals of the rhombus measure 7 inches and 9 inches, how much space is available for engraving text onto the award? (Lesson 11-2)

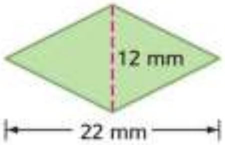
7. **MULTIPLE CHOICE** The area of a kite is 4 square feet. If the tail is to be 3 times longer than the kite's long diagonal, and the short diagonal measures 2 feet, how long should the kite's tail be? (Lesson 11-2)

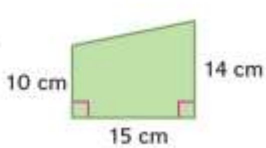
   A 4 feet
   B 6 feet
   C 7 feet
   D 12 feet

Find the area of each trapezoid, rhombus, or kite. (Lesson 11-2)

8. 

9. 

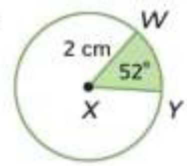
10. 

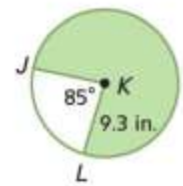
11. 

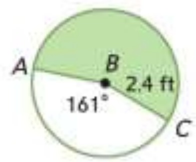
12. **ARCHAEOLOGY** The most predominant shape in Incan architecture is the trapezoid. The doorway pictured below is 3 feet wide at the top and 4 feet wide at the bottom. A person who is 5 feet 8 inches tall can barely pass through the doorway. How much fabric would be necessary to make a curtain for the doorway? (Lesson 11-2)

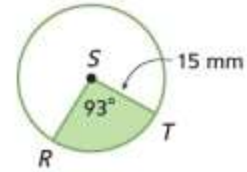
13. **ALGEBRA** A sector of a circle has a central angle measure of 30° and radius r. Write an expression for the perimeter of the sector in terms of r. (Lesson 11-3)

Find the area of each shaded sector. Round to the nearest tenth. (Lesson 11-3)

14. 

15. 

16. 

17. 

Find the indicated measure. Round to the nearest tenth. (Lesson 11-3)

18. The area of a circle is 52 square inches. Find the diameter.

19. Find the radius of a circle with an area of 104 square meters.

20. **FRUIT** The diameter of the orange slice shown is 9 centimeters. If each of the orange's 10 sections are congruent, find the approximate area covered by 8 sections. (Lesson 11-3)

EXPLORE

11-4 Geometry Lab
Investigating Areas of Regular Polygons

The point in the interior of a regular polygon that is equidistant from all of the vertices is the *center* of the polygon. A segment from the center that is perpendicular to a side of the polygon is an **apothem**.

Mathematical Processes

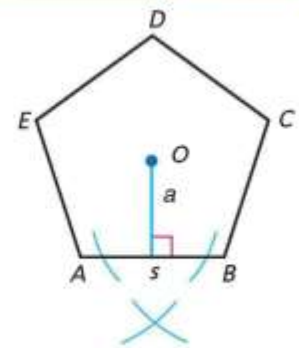
MP **G.1(C)** Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

Activity

Work cooperatively.

Step 1 Copy regular pentagon *ABCDE* and its center *O*.

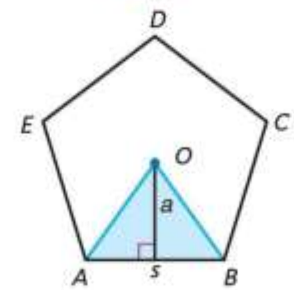
Step 2 Draw the apothem from *O* to side $\overline{AB}$ by constructing the perpendicular bisector of $\overline{AB}$. Label the apothem measure as *a*. Label the measure of $\overline{AB}$ as *s*.



Step 3 Use a straightedge to draw $\overline{OA}$ and $\overline{OB}$.

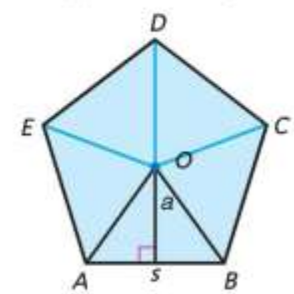
Step 4 What measure in $\triangle AOB$ represents the base of the triangle? What measure represents the height?

Step 5 Find the area of $\triangle AOB$ in terms of *s* and *a*.



Step 6 Draw $\overline{OC}$, $\overline{OD}$, and $\overline{OE}$. What is true of the five small triangles formed?

Step 7 How do the areas of the five triangles compare?



Analyze the Results

Work cooperatively.

1. The area of a pentagon *ABCDE* can be found by adding the areas of the given triangles that make up the pentagonal region.

   $A = \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa$

   $A = \frac{1}{2}(sa + sa + sa + sa + sa)$ or $\frac{1}{2}(5sa)$

   What does $5s$ represent?

2. Write a formula for the area of a pentagon in terms of perimeter *P*.

LESSON 11-4 Areas of Regular Polygons and Composite Figures

Then

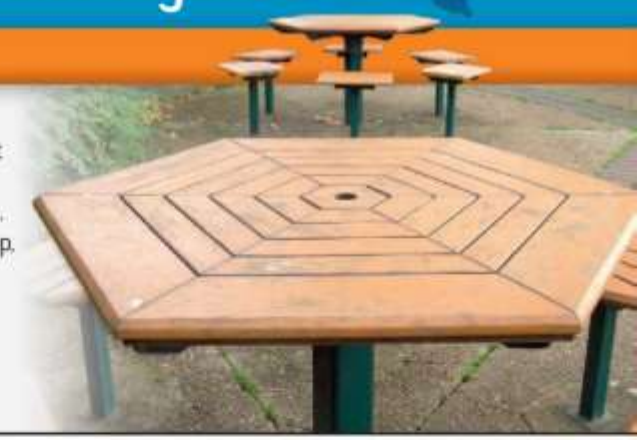
- You used inscribed and circumscribed figures and found the areas of circles.

Now

1. Find areas of regular polygons.
2. Find areas of composite figures.

Why?

- The top of the table shown is a regular hexagon. Notice that the top is composed of six congruent triangular sections. To find the area of the table top, you can find the sum of the areas of the sections.



Targeted TEKS

G.11(A) Apply the formula for the area of regular polygons to solve problems using appropriate units of measure.

G.11(B) Determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure.
Also addresses G.10(B).

Mathematical Processes

G.1(E) Create and use representations to organize, record, and communicate mathematical ideas.
Also addresses G.1(B).

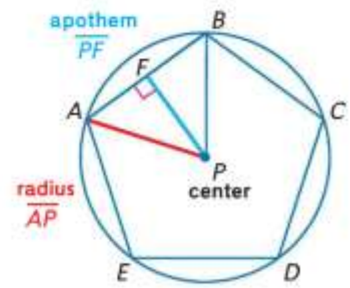
New Vocabulary

center of a regular polygon
radius of a regular polygon
apothem
central angle of a regular polygon
composite figure

1 Areas of Regular Polygons

In the figure, a regular pentagon is *inscribed* in ⊙P, and ⊙P is *circumscribed* about the pentagon. The **center of a regular polygon** and the **radius of a regular polygon** are also the center and the radius of its circumscribed circle.

A segment drawn from the center of a regular polygon perpendicular to a side of the polygon is called an **apothem**. Its length is the height of an isosceles triangle that has two radii as legs.

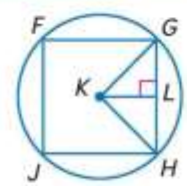


$\angle APB$ is a central angle of regular pentagon *ABCDE*.

A **central angle of a regular polygon** has its vertex at the center of the polygon and its sides pass through consecutive vertices of the polygon. The measure of each central angle of a regular n-gon is $\frac{360}{n}$.

Example 1 Identify Segments and Angles in Regular Polygons

Square *FGHJ* is inscribed in ⊙*K*. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.



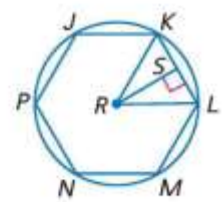
center: point K | radius: $\overline{KG}$ or $\overline{KH}$

apothem: $\overline{KL}$ | central angle: $\angle GKH$

A square is a regular polygon with 4 sides. Thus, the measure of each central angle of square *FGHJ* is $\frac{360}{4}$ or 90.

Guided Practice

1. In the figure, regular hexagon *JKLMNP* is inscribed in ⊙R. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.



You can find the area of any regular n-gon by dividing the polygon into congruent isosceles triangles. This strategy is sometimes called *decomposing the polygon into triangles*.

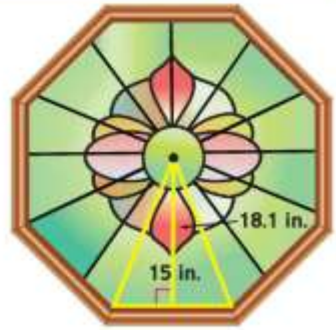
TEKS G.11(A)

Real-World Example 2 Area of a Regular Polygon

ReadingMath

Apothem Like the *radius* of a circle, the *apothem* of a polygon refers to the length of any apothem of the polygon.

ART Kang created the stained glass window shown. The window is a regular octagon with a side length of 15 inches and an apothem of 18.1 inches. What is the area covered by the window?



Step 1 Divide the polygon into congruent isosceles triangles.

Since the polygon has 8 sides, the polygon can be divided into 8 congruent isosceles triangles, each with a base of 15 inches and a height of 18.1 inches.

Step 2 Find the area of one triangle.

$A = \frac{1}{2}bh$	Area of a triangle
$= \frac{1}{2}(15)(18.1)$	$b = 15$ and $h = 18.1$
$= 135.75 \text{ in}^2$	Simplify.

Step 3 Multiply the area of one triangle by the total number of triangles.

Since there are 8 triangles, the area of the stained glass is $135.75 \cdot 8$ or 1086 square inches.

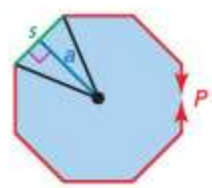
GuidedPractice

2. **HOT TUBS** The cover of the hot tub shown is a regular pentagon. If the side length is 2.5 feet and the apothem is 1.7 feet, find the area of the lid to the nearest tenth.

WatchOut!

Area of Regular Polygon this approach can only be applied to *regular* polygons.

From Example 2, we can develop a formula for the area of a regular n-gon with side length s and apothem a.

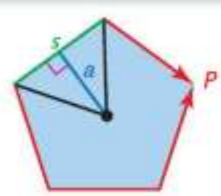


$A =$ area of one triangle $\cdot$ number of triangles	
$= \frac{1}{2} \cdot$ base $\cdot$ height $\cdot$ number of triangles	
$= \frac{1}{2} \cdot s \cdot a \cdot n$	Base of triangle is s and height is a. The number of triangles is n.
$= \frac{1}{2} \cdot a \cdot (n \cdot s)$	Commutative and Associative Properties
$= \frac{1}{2} \cdot a \cdot P$	The perimeter P of the polygon is $n \cdot s$.

KeyConcept Area of a Regular Polygon

Words The area A of a regular n-gon with side length s is one half the product of the apothem a and perimeter P.

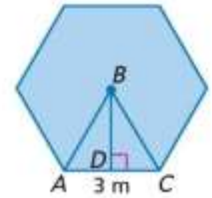
Symbols $A = \frac{1}{2}a(ns)$ or $A = \frac{1}{2}aP$.



Example 3 Use the Formula for the Area of a Regular Polygon

Find the area of each regular polygon. Round to the nearest tenth.

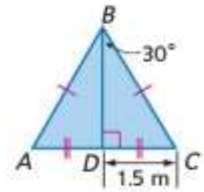
a. regular hexagon



Step 1 Find the measure of a central angle.

A regular hexagon has 6 congruent central angles, so $m\angle ABC = \frac{360}{6}$ or 60.

Step 2 Find the apothem.



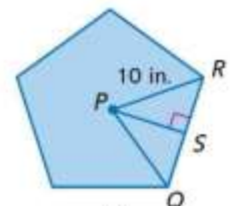
Apothem $\overline{BD}$ is the height of isosceles $\triangle ABC$. It bisects $\angle ABC$, so $m\angle DBC = 30$. It also bisects $\overline{AC}$, so $DC = 1.5$ meters.

$\triangle BDC$ is a 30°-60°-90° triangle with a shorter leg that measures 1.5 meters, so $BD = 1.5\sqrt{3}$ meters.

Step 3 Use the apothem and side length to find the area.

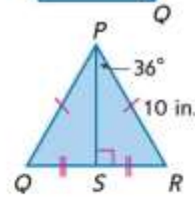
$A = \frac{1}{2}aP$	Area of a regular polygon
$= \frac{1}{2}(1.5\sqrt{3})(18)$	$a = 1.5\sqrt{3}$ and $P = 6(3)$ or 18
$\approx 23.4\text{ m}^2$	Use a calculator.

b. regular pentagon



Step 1 A regular pentagon has 5 congruent central angles, so $m\angle QPR = \frac{360}{5}$ or 72.

Step 2 Apothem $\overline{PS}$ is the height of isosceles $\triangle RPQ$. It bisects $\angle RPQ$, so $m\angle RPS = 36$. Use trigonometric ratios to find the side length and apothem of the polygon.



$\sin 36° = \frac{SR}{10}$ $\qquad$ $\cos 36° = \frac{PS}{10}$

$10 \sin 36° = SR$ $\qquad$ $10 \cos 36° = PS$

$QR = 2SR$ or $2(10 \sin 36°)$. So the pentagon's perimeter is $5 \cdot 2(10 \sin 36°)$ or $10(10 \sin 36°)$. The length of the apothem $\overline{PS}$ is $10 \cos 36°$.

Step 3

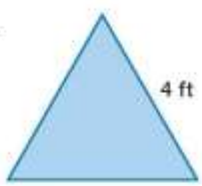
$A = \frac{1}{2}aP$	Area of a regular polygon
$= \frac{1}{2}(10 \cos 36°)[10(10 \sin 36°)]$	$a = 10 \cos 36°$, $P = 10(10 \sin 36°)$
$\approx 237.8\text{ in}^2$	Use a calculator.

StudyTip

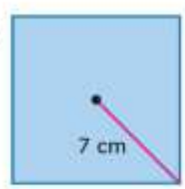
Apply Math The altitude of an isosceles triangle from its vertex to its base is also an angle bisector and median of the triangle.

GuidedPractice

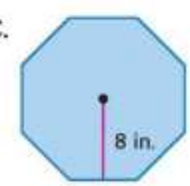
3A.



3B.



3C.



2 Areas of Composite Figures

A **composite figure** is a figure that can be separated into regions that are basic figures, such as triangles, rectangles, trapezoids, and circles. To find the area of a composite figure, find the area of each basic figure and then use the Area Addition Postulate.

Real-WorldLink

The first miniature golf course was built in Pinehurst, North Carolina, on a private estate owned by James Barber. There are currently between 5000 and 7500 miniature golf courses in the United States.

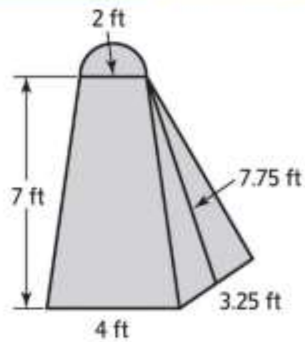
Source: Miniature Golf Association of the United States

TEKS G.11(B)

Example 4 Find the Area of a Composite Figure by Adding

When viewed from above, the putting green at a miniature golf course is composed of a semicircle, trapezoid, and triangle. Which of the following best represents the area of carpet needed to cover the green?

A 21 **B** 32 **C** 35 **D** 37



Read the Item

The area to be carpeted can be separated into a trapezoid with a height of 7 feet and bases of 2 feet and 4 feet, a triangle with a base of 3.25 feet and height of 7.75 feet, hypotenuse of 5.7 feet, and a semicircle with a diameter of 2 feet. Find the area of each figure separately and add to get the total area.

Solve the Item

Area of green = **area of trapezoid** + **area of triangle** + **area of semicircle.**

$= \frac{1}{2} \cdot h\,(b_1 + b_2) \quad + \frac{1}{2} \cdot b \cdot h \quad + \frac{1}{2}\pi r^2$

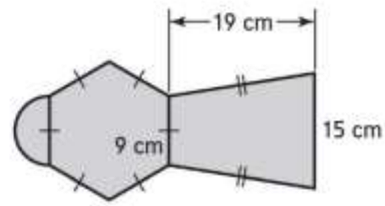
$\approx \frac{1}{2} \cdot 7 \cdot (2 + 4) \quad + \frac{1}{2} \cdot 3.25 \cdot 7.75 \quad + \frac{1}{2}\pi(1)^2$

$\approx 21 + 12.59 + 1.57$ or about 35.16 ft^2

So, about 35 square feet of carpet is needed. The correct answer is C.

GuidedPractice

The figure shown is composed of a semicircle, regular hexagon, and trapezoid. What is the area of the figure?



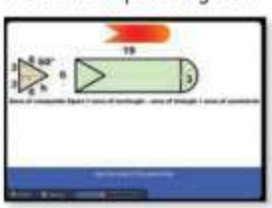
F 427.8 cm^2 **H** 454.3 cm^2

G 438.4 cm^2 **J** 470.2 cm^2

The areas of some figures can be found by subtracting the areas of basic figures.

Go Online!

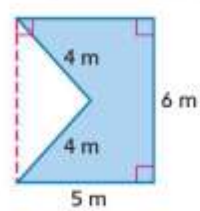
Log into ConnectED to watch an **Animation** about areas of composite figures.



TEKS G.11(B)

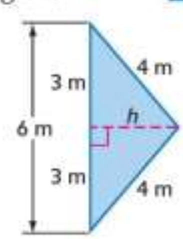
Example 5 Find the Area of a Composite Figure by Subtracting

Find the area of the figure. Round to the nearest tenth if necessary.



To find the area of the figure, subtract the area of the triangle from the area of the rectangle.

Using the Pythagorean Theorem, the height h of the triangle is $\sqrt{4^2 - 3^2}$ or $\sqrt{7}$ meters.



Area of figure = **Area of rectangle** − **Area of triangle**

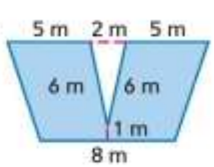
$= \quad b \cdot h \quad - \quad \frac{1}{2}bh$

$= \quad 5 \cdot 6 \quad - \quad \frac{1}{2}(6)(\sqrt{7})$

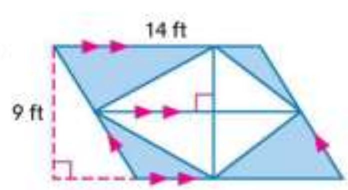
$\approx 30 - 7.9$ or about 22.1 m^2

GuidedPractice

5A.



5B.



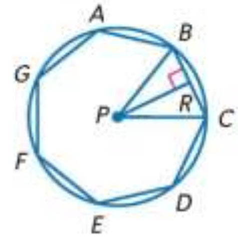
Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Example 1

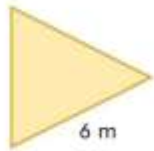
1. In the figure, heptagon $ABCDEFG$ is inscribed in $\odot P$. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.



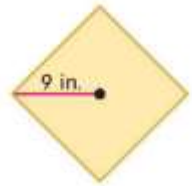
Examples 2–3 TEKS G.11(A)

Find the area of each regular polygon. Round to the nearest tenth.

2.

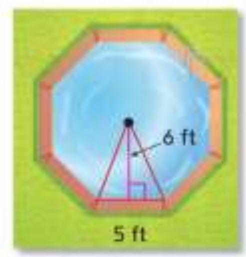


3.



4.

5. **POOLS** Kenton's job is to cover the community pool during fall and winter. Since the pool is in the shape of an octagon, he needs to find the area in order to have a custom cover made. If the pool has the dimensions shown at the right, what is the area of the pool?



Example 4 TEKS G.11(B)

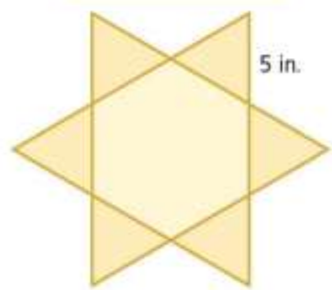
6. **MULTIPLE CHOICE** The figure shown is composed of a regular hexagon and equilateral triangles. Which of the following best represents the area?

A 37.5 in^2

B $37.5\sqrt{3}$ in^2

C 75 in^2

D $75\sqrt{3}$ in^2



Example 5 TEKS G.11(B)

7 **BASKETBALL** The basketball court in Jeff's school is painted as shown.

a. What area of the court is blue? Round to the nearest square foot.

b. What area of the court is red? Round to the nearest square foot.

Note: Art not drawn to scale.

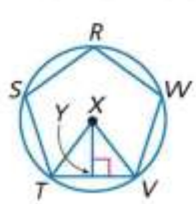
Practice and Problem Solving

Extra Practice is on page R11.

Example 1

MP **ORGANIZE IDEAS** **In each figure, a regular polygon is inscribed in a circle. Identify the center, a radius, an apothem, and a central angle of each polygon. Then find the measure of a central angle.**

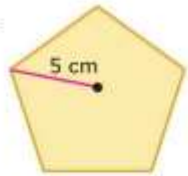
8.

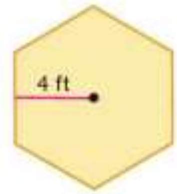


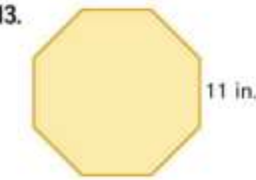
9.

Examples 2–3 Find the area of each regular polygon. Round to the nearest tenth.
TEKS G.11(A)

10.

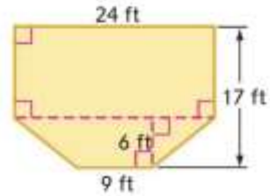

11.


12.


13.


Example 4
TEKS G.11(B)

14. **CARPETING** Ignacio's family is getting new carpet in their family room, and they want to determine how much the project will cost.

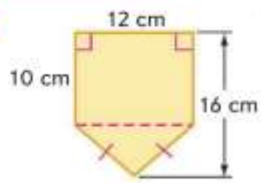


a. Use the floor plan shown to find the area to be carpeted.

b. If the carpet costs $4.86 per square yard, how much will the project cost?

Examples 4–5
TEKS G.11(B)

MP **ORGANIZE IDEAS** Find the area of each figure. Round to the nearest tenth if necessary.

15.


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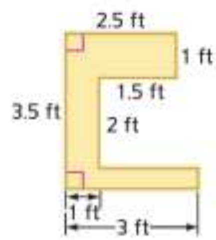

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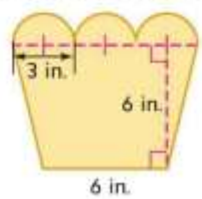

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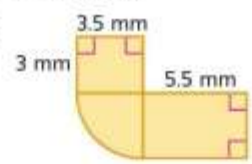
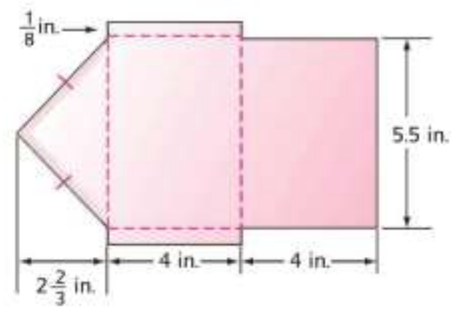


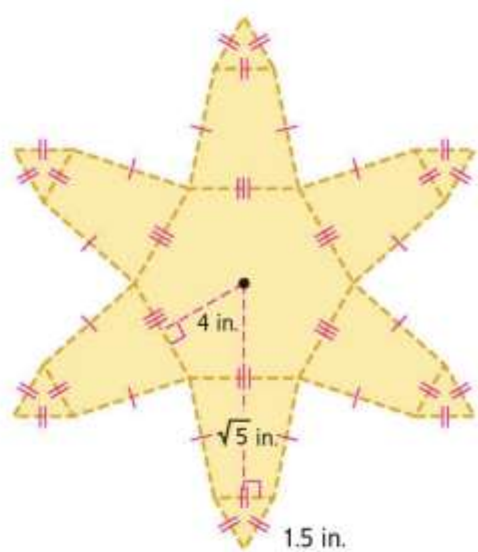
19. **CRAFTS** Latoya's greeting card company is making envelopes for a card from the pattern shown.



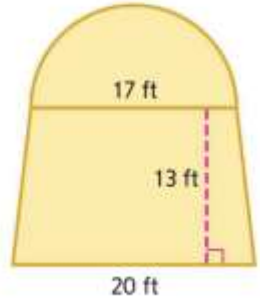
a. Find the perimeter and area of the pattern. Round to the nearest tenth.

b. If Latoya orders sheets of paper that are 2 feet by 4 feet, how many envelopes can she make per sheet?

20. **VOLUNTEERING** James is making pinwheels at a summer camp. If they want to paint one side of each pinwheel, find the approximate total area of 10 pinwheels.

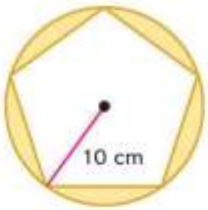


21. **THEATRE** Alison's drama club is planning on painting the amphitheater stage. Find the total area of the stage.

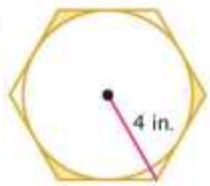


Find the area of each shaded region formed by each circle and regular polygon. Round to the nearest tenth.

22.



23.



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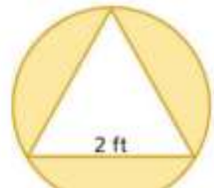
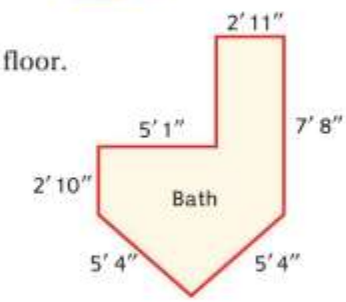


25. **FLOORING** JoAnn wants to lay 12″ × 12″ tile on her bathroom floor.

a. Find the area of the bathroom floor in her apartment floor plan.

b. If the tile comes in boxes of 15 and JoAnn buys no extra tile, how many boxes will she need?



Find the perimeter and area of each figure. Round to the nearest tenth, if necessary.

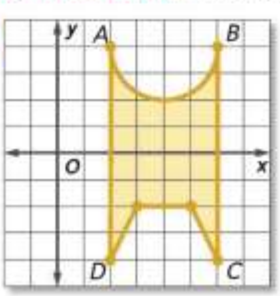
26. a regular hexagon with a side length of 12 centimeters

27. a regular pentagon circumscribed about a circle with a radius of 8 millimeters

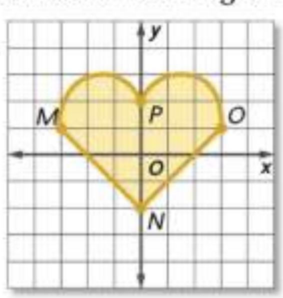
28. a regular octagon inscribed in a circle with a radius of 5 inches

MP **ORGANIZE IDEAS** **Find the area of each shaded region. Round to the nearest tenth.**

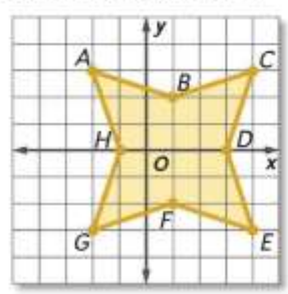
29.



30.



31.

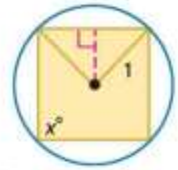


32. Find the total area of the shaded regions. Round to the nearest tenth.

TEKS G.10(B)

33. **CHANGING DIMENSIONS** Calculate the area of an equilateral triangle with a perimeter of 3 inches. Calculate the areas of a square, a regular pentagon, and a regular hexagon with perimeters of 3 inches. How does the area of a regular polygon with a fixed perimeter change as the number of sides increases?

34. **MP** **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the areas of regular polygons inscribed in circles.



a. Geometric Draw a circle with a radius of 1 unit and inscribe a square. Repeat twice, inscribing a regular pentagon and hexagon.

b. Algebraic Use the inscribed regular polygons from part **a** to develop a formula for the area of an inscribed regular polygon in terms of angle measure x and number of sides n.

c. Tabular Use the formula you developed in part **b** to complete the table below. Round to the nearest hundredth.

Number of Sides, n	4	5	6	8	10	20	50	100
Interior Angle Measure, x								
Area of Inscribed Regular Polygon								

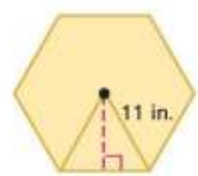
d. Verbal Make a conjecture about the area of an inscribed regular polygon with a radius of 1 unit as the number of sides increases.

TEKS G.11(A), G.11(B)

H.O.T. Problems Use Higher-Order Thinking Skills

35. **ERROR ANALYSIS** Chloe and Flavio want to find the area of the hexagon shown. Is either of them correct? Explain your reasoning.

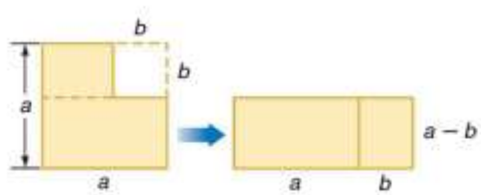
Chloe	Flavio
$A = \frac{1}{2}Pa$	$A = \frac{1}{2}Pa$
$= \frac{1}{2}(66)(9.5)$	$= \frac{1}{2}(33)(9.5)$
$= 313.5 \text{ in}^2$	$= 156.8 \text{ in}^2$



36. **MP** **PROBLEM SOLVING** Using the map of Nevada shown, estimate the area of the state. Explain your reasoning.

37. **MP** **ORGANIZE IDEAS** Draw a pair of composite figures that have the same area. Make one composite figure out of a rectangle and a trapezoid, and make the other composite figure out of a triangle and a rectangle. Show the area of each basic figure.

38. **WRITING IN MATH** Consider the sequence of area diagrams shown.



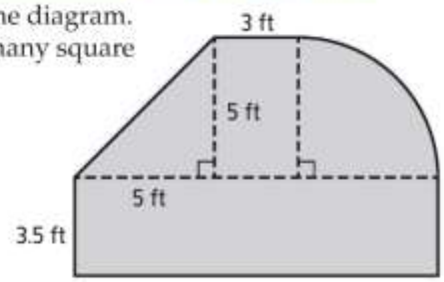
a. What algebraic theorem do the diagrams prove? Explain your reasoning.

b. Create your own sequence of diagrams to prove a different algebraic theorem.

39. **WRITING IN MATH** How can you find the area of any figure?

Example TEKS G.11(B) MP G.1(A)

TEKS REVIEW The dimensions of a patio are shown in the diagram. If the surface of the patio is to be painted, about how many square feet will be painted?



A 66.8 ft^2

B 92.6 ft^2

C 112.3 ft^2

D 151.5 ft^2

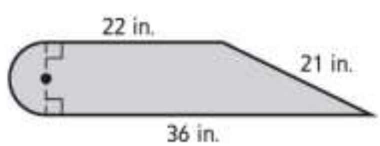
This is a composite figure that can be separated into an isosceles right triangle with legs of 5 feet, two rectangles: 3 feet by 5 feet and 3.5 feet by 13 feet, and a quarter circle with a radius of 5 feet. To find the total area, find the sum of the areas of each shape.

Area of patio = Area of triangle + Areas of rectangles + Area of quarter circle

$$= \frac{1}{2}bh + \ell_1 w_1 + \ell_2 w_2 + \frac{1}{4}\pi r^2$$
$$= \frac{1}{2}(5 \cdot 5) + 3 \cdot 5 + 3.5 \cdot 13 + \frac{1}{4}\pi(5^2)$$
$$= 12.5 + 15 + 45.5 + 6.25\pi$$
$$\approx 92.6 \text{ ft}^2$$

Approximately 92.6 square feet will be painted. The correct answer is choice B.

40. **ACT/SAT** Which of the following is the best estimate of the area of the composite figure shown here?



A 550 in^2

B 646 in^2

C 660 in^2

D 782 in^2

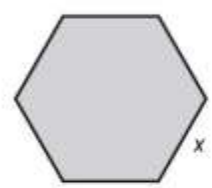
E 839 in^2

41. What is the area of a square with an apothem of 2 feet? TEKS G.11(A) MP G.1(E), G.1(F)

F 16 ft^2 H 4 ft^2

G 8 ft^2 J 2 ft^2

42. A regular hexagon has sides that are x units long.

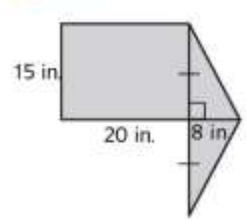


Which of the following expressions represents the area of the hexagon in square units? TEKS G.11(A) MP G.1(B), G.1(F)

A $\frac{3\sqrt{2}}{2}x^2$ C $3\sqrt{3}\,x^2$

B $\frac{\sqrt{3}}{4}x^2$ D $\frac{3\sqrt{3}}{2}x^2$

43. **GRIDDABLE** Find the area of the shaded figure in square inches. Round to the nearest tenth. TEKS G.11(B) MP G.1(E)

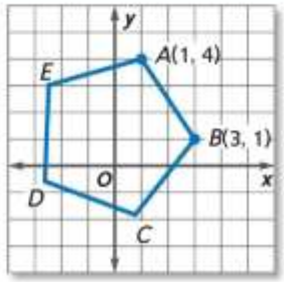


EXTEND

11-4 Geometry Lab
Regular Polygons on the Coordinate Plane

If you know the coordinates of two consecutive vertices of a regular polygon, you can use the Distance Formula to find the length of each side. For example, in the figure shown, the length of $\overline{AB}$ is $\sqrt{(3-1)^2+(1-4)^2}$ or $\sqrt{13}$. Using this measure, you can then find the perimeter and area of the figure using the techniques presented in Lesson 11-4.

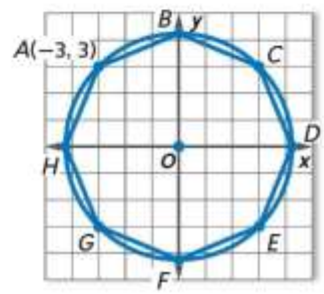
You can also use the Distance Formula to find the perimeter and area of a regular polygon inscribed in a circle given the coordinates of the endpoints of a radius.



Targeted TEKS
G.11(A) Apply the formula for the area of regular polygons to solve problems using appropriate units of measure.

Activity 1 Inscribed Polygon

Work cooperatively. Find the perimeter and area of octagon *ABCDEFGH*, which is inscribed in ⊙*O*. Round to the nearest tenth, if necessary.

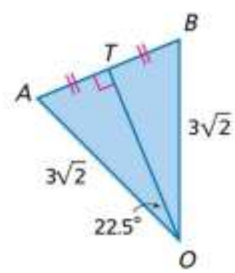


Step 1 Use the Distance Formula to find a radius of ⊙*O*.

$OA = \sqrt{(-3-0)^2+(3-0)^2}$ — $x_2 = -3, x_1 = 0, y_2 = 3$, and $y_1 = 0$

$= \sqrt{18}$ or $3\sqrt{2}$ — Simplify.

Step 2 Find the perimeter and area.



Because the octagon is inscribed in ⊙*O*, $\overline{OA}$ and $\overline{OB}$ are both radii of ⊙*O*. Therefore, $OA = OB = 3\sqrt{2}$. Let $\overline{OT}$ be an apothem of the octagon with length a. Then $\overline{OT}$ is also the height of isosceles $\triangle AOB$. Since the octagon is regular, $m\angle AOB$ is $360 \div 8$ or 45. Since $\overline{OT}$ bisects $\angle AOB$ and side $\overline{AB}$, $m\angle AOT = 45 \div 2$ or 22.5, and $AB = 2(AT)$.

Use trigonometric ratios to find a and AT.

$\cos 22.5° = \frac{a}{3\sqrt{2}}$ — $\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$a = 3\sqrt{2}\cos 22.5°$ — Solve for a.

$\sin 22.5° = \frac{AT}{3\sqrt{2}}$ — $\sin\theta = \frac{\text{opp}}{\text{hyp}}$

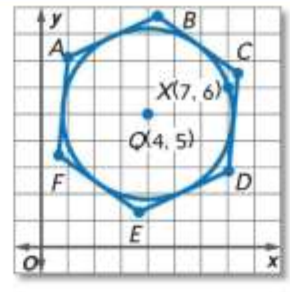
$AT = 3\sqrt{2}\sin 22.5°$ — Solve for AT.

$AB = 2(AT)$, so $AB = 2(3\sqrt{2}\sin 22.5°)$ and the perimeter P of the octagon is $8(2)3\sqrt{2}\sin 22.5°$ or about 26.0 units. The area of the octagon is $\frac{1}{2}aP$, which is $\frac{1}{2}3\sqrt{2}\cos 22.5° \cdot 8(2)3\sqrt{2}\sin 22.5°$ or about 50.9 units^2.

You can also use the Distance Formula to find the perimeter and area of a regular polygon circumscribed about a circle given the coordinates of the endpoints of a radius.

Activity 2 Circumscribed Polygon

Work cooperatively. Find the perimeter and area of hexagon $ABCDEF$, which is circumscribed about $\odot Q$. Round to the nearest tenth, if necessary.

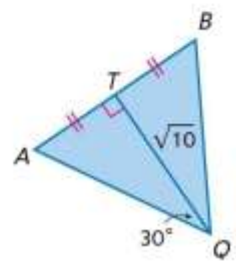


Step 1 Use the Distance Formula to find a radius of $\odot Q$.

$QX = \sqrt{(7-4)^2 + (6-5)^2}$ or $\sqrt{10}$ $\quad$ $x_2 = 7, x_1 = 4, y_2 = 6$, and $y_1 = 5$

Step 2 Find the perimeter and area of hexagon $ABCDEF$.

Because the hexagon is circumscribed about $\odot Q$, $\overline{AB}$ is tangent to the circle. Let the point of tangency be T. Since all radii of a circle are congruent, radius $\overline{QT}$ also measures $\sqrt{10}$. $\overline{QT}$ is an apothem of the hexagon, so $a = \sqrt{10}$.



The apothem is also the height of isosceles $\triangle AQB$. Since the hexagon is regular, $m\angle AQB$ is $360 \div 6$ or 60. Since $\overline{QT}$ bisects $\angle AQB$ and side $\overline{AB}$, $m\angle AQT = 60 \div 2$ or 30, and $AB = 2(AT)$. Use trigonometric ratios to find AT. Then find AB.

$\tan 30° = \frac{AT}{\sqrt{10}}$ $\quad$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

$AT = \sqrt{10}\tan 30°$ $\quad$ Solve for AT.

$AT = \sqrt{10}\left(\frac{\sqrt{3}}{3}\right)$ or $\frac{\sqrt{30}}{3}$ $\quad$ $\tan 30° = \frac{\sqrt{3}}{3}$

$AB = 2(AT)$

$= 2\left(\frac{\sqrt{30}}{3}\right)$

$= \frac{2\sqrt{30}}{3}$

The perimeter P of the hexagon is $6 \cdot \frac{2\sqrt{30}}{3}$ or $4\sqrt{30}$, which is about 21.9 units. The area of the hexagon is $\frac{1}{2}aP$, which is $\frac{1}{2}\sqrt{10}(4\sqrt{30})$ or about 34.6 units^2.

Exercises

Work cooperatively. Find the perimeter and area of each regular polygon with the given consecutive vertices. Round to the nearest tenth, if necessary.

1. pentagon $ABCDE$; $A(1, 4)$, $B(3, 1)$

2. hexagon $ABCDEF$; $A(-4, 2)$, $B(0, 5)$

Find the perimeter and area of each regular polygon inscribed in $\odot O$, centered at the origin, and containing the given point. Round to the nearest tenth, if necessary.

3. pentagon $ABCDE$; $E(-4, -1)$

4. hexagon $ABCDEF$; $D(4, -5)$

Find the perimeter and area of each regular polygon circumscribed about $\odot Q$, with the given center and point X on the circle. Round to the nearest tenth, if necessary.

5. pentagon $ABCDE$; $Q(-2, 1)$; $X(-1, 3)$

6. octagon $ABCDEFGH$; $Q(3, -1)$; $X(1, -3)$

LESSON 11-5 Areas of Similar Figures

Then

- You used scale factors and proportions to solve problems involving the perimeters of similar figures.

Now

- 1 Find areas of similar figures by using scale factors.
- 2 Determine how changes in dimensions affect the areas of figures.

Why?

- Architecture firms often hire model makers to make scale models of projects that are used to sell their designs. Since the base of a model is geometrically similar to the base of the actual building it represents, their areas are related.

Targeted TEKS

G.10(B) Determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.

Mathematical Processes

G.1(A) Apply mathematics to problems arising in everyday life, society, and the workplace.

G.1(F) Analyze mathematical relationships to connect and communicate mathematical ideas.

1 Areas of Similar Figures

In Lesson 7-2, you learned that if two polygons are similar, then their perimeters are proportional to the scale factor between them. The areas of two similar polygons share a different relationship.

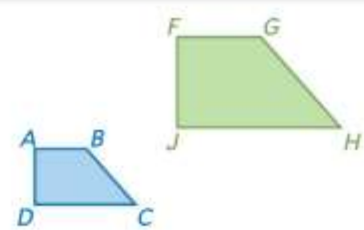
$$\frac{\text{perimeter of figure } B}{\text{perimeter of figure } A} = \frac{28k}{28} \text{ or } k$$

$$\frac{\text{area of figure } B}{\text{area of figure } A} = \frac{45k^2}{45} \text{ or } k^2$$

Theorem 11.1 Areas of Similar Polygons

Words If two polygons are similar, then their areas are proportional to the square of the scale factor between them.

Example If $ABCD \sim FGHJ$, then $\frac{\text{area of } FGHJ}{\text{area of } ABCD} = \left(\frac{FG}{AB}\right)^2$.

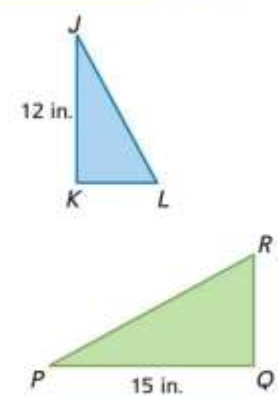


You will prove Theorem 11.1 for triangles in Exercise 22.

TEKS G.10(B)

Example 1 Find Areas of Similar Polygons

If $\triangle JKL \sim \triangle PQR$ and the area of $\triangle JKL$ is 30 square inches, find the area of $\triangle PQR$.



The scale factor between $\triangle PQR$ and $\triangle JKL$ is $\frac{15}{12}$ or $\frac{5}{4}$, so the ratio of their areas is $\left(\frac{5}{4}\right)^2$.

$\frac{\text{area of } \triangle PQR}{\text{area of } \triangle JKL} = \left(\frac{5}{4}\right)^2$	Write a proportion.
$\frac{\text{area of } \triangle PQR}{30} = \frac{25}{16}$	Area of $\triangle JKL = 30$ and $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$
area of $\triangle PQR = \frac{25}{16} \cdot 30$	Multiply each side by 30.
area of $\triangle PQR = 46.875$	Simplify.

So the area of $\triangle PQR$ is about 46.9 square inches.

Nicholas Cope/Stone/Getty Images

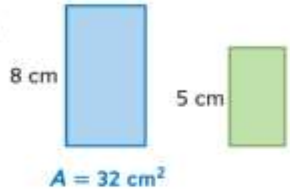
Go Online!

Discover a relationship between the areas of similar figures with a **Geometer's Sketchpad®** sketch in ConnectED.

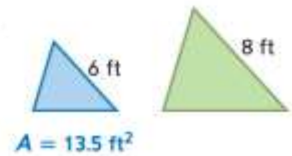
GuidedPractice

For each pair of similar figures, find the area of the green figure.

1A.



1B.

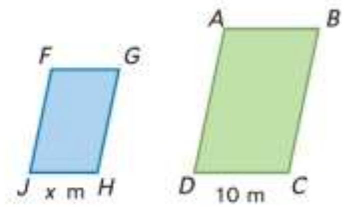


You can use the areas of similar figures to find the scale factor between them or a missing measure.

TEKS G.10(B)

Example 2 Use Areas of Similar Figures

The area of ▱$ABCD$ is 150 square meters. The area of ▱$FGHJ$ is 54 square meters. If ▱$ABCD \sim$ ▱$FGHJ$, find the scale factor of ▱$FGHJ$ to ▱$ABCD$ and the value of x.



Let k be the scale factor between ▱$FGHJ$ and ▱$ABCD$.

$\frac{\text{area of ▱}FGHJ}{\text{area of ▱}ABCD} = k^2$ Theorem 11.1

$\frac{54}{150} = k^2$ Substitution

$\frac{9}{25} = k^2$ Simplify.

$\frac{3}{5} = k$ Take the positive square root of each side.

So the scale factor of ▱$FGHJ$ to ▱$ABCD$ is $\frac{3}{5}$. Use this scale factor to find the value of x.

$\frac{JH}{DC} = k$ The ratio of corresponding lengths of similar polygons is equal to the scale factor between the polygons.

$\frac{x}{10} = \frac{3}{5}$ Substitution

$x = \frac{3}{5} \cdot 10$ or 6 Multiply each side by 10.

CHECK Confirm that $\frac{JH}{DC}$ is equal to the scale factor.

$\frac{JH}{DC} = \frac{6}{10} = \frac{3}{5}$ ✓

WatchOut!

Writing Ratios When finding the ratio of the area of Figure A to the area of Figure B, be sure to write your ratio as $\frac{\text{area of figure } A}{\text{area of figure } B}$.

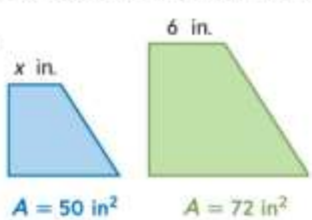
ReadingMath

Ratios Ratios can be written in different ways. For example, x to y, $x : y$, and $\frac{x}{y}$ are all representations of the ratio of x and y.

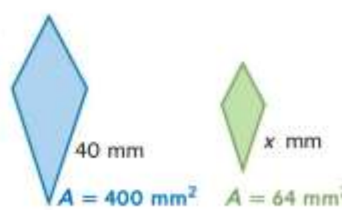
GuidedPractice

For each pair of similar figures, use the given areas to find the scale factor of the blue to the green figure. Then find x.

2A.



2B.



2 Dimensional Changes

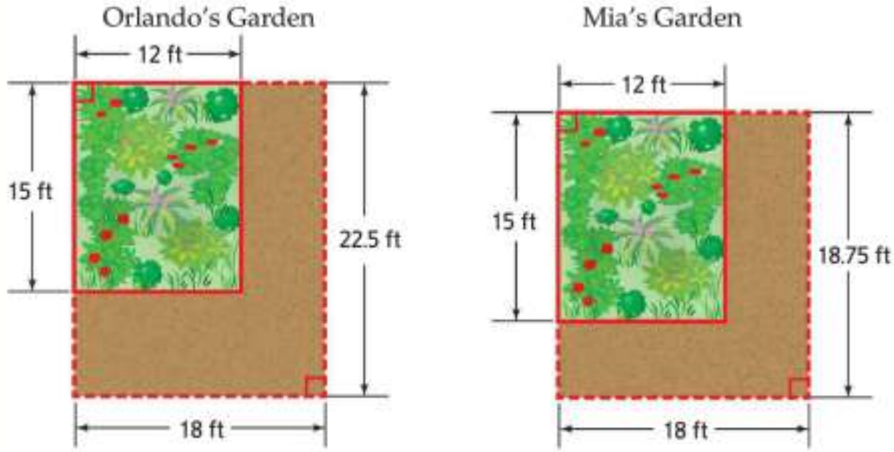
When the dimensions of a figure are changed proportionally, the new figure is similar to the original figure. Changing the dimensions nonproportionally does not result in similar figures.

TEKS G.10(B)

Real-World Example 3 Changing Dimensions

GARDENING Orlando and Mia each have 12-feet by 15-feet rectangular gardens which they plan to expand. Orlando's new garden will measure 18 feet by 22.5 feet while Mia's new garden will measure 18 feet by 18.75 feet. Describe how the changes in dimensions affect the areas of each garden.

Draw a diagram and label the measurements.



Next, compare the new dimensions to the original dimensions to determine whether the increases are proportional or nonproportional.

Orlando's Garden		Mia's Garden
$\frac{18 \text{ ft}}{12 \text{ ft}} = \frac{3}{2}$	$\frac{\text{new width}}{\text{original width}}$	$\frac{18 \text{ ft}}{12 \text{ ft}} = \frac{3}{2}$
$\frac{22.5 \text{ ft}}{15 \text{ ft}} = \frac{3}{2}$	$\frac{\text{new length}}{\text{original length}}$	$\frac{18.75 \text{ ft}}{15 \text{ ft}} = \frac{5}{4}$

Since each dimension of Orlando's garden increased by the same scale factor this is a proportional dimension change. So the original garden and new garden are similar figures. By Theorem 11.1, the ratio of the areas is the square of the scale factor.

$$\frac{\text{new area}}{\text{original area}} = \frac{(18)(22.5)}{(12)(15)} = \frac{405}{180} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$$

Since different scale factors were used for each dimension, Mia's garden is not changing proportionally. Compute the ratio of the new area to the original using the scale factors applied to each dimension.

$$\frac{\text{new area}}{\text{original area}} = \frac{(12 \cdot 1.5)(15 \cdot 1.25)}{(12)(15)} = 1.5 \cdot 1.25 \text{ or } 1.875$$

Notice that the ratio of the new area to the original area is the product of the scale factors used to enlarge the dimensions.

Orlando's garden is increasing proportionally so the ratio of the new area to the original area is the square of the scale factor. Mia's garden is not increasing proportionally. The ratio of the new area to the original area is the product of the scale factors.

Guided Practice

3. **CRAFTS** Miyoki is crocheting two circles. If the diameter of the smaller circle is about 8 centimeters and the diameter of the larger circle is about 12.6 centimeters, describe how the difference in dimensions affects the areas of the circles.

Reading Math

Similar Circles Since all circles have the same shape, all circles are similar. Therefore, the areas of two circles are also related by the square of the scale factor between them.

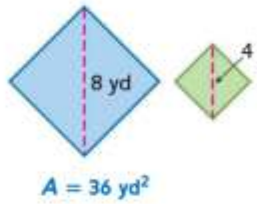
Check Your Understanding

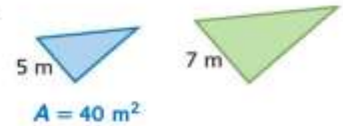
= Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Example 1 TEKS G.10(B)

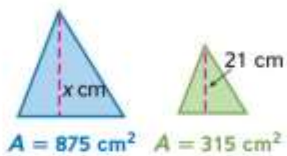
For each pair of similar figures, find the area of the green figure.

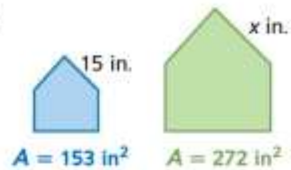
1\. 

2\. 

Example 2 TEKS G.10(B)

For each pair of similar figures, use the given areas to find the scale factor from the blue to the green figure. Then find x.

3\. 

4\. 

Example 3 TEKS G.10(B)

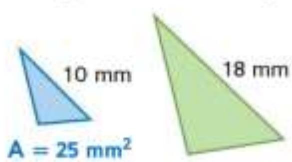
5\. **MEMORIES** Zola has a picture frame that holds all of her school pictures. Each small opening is similar to the large opening in the center. If the center opening has an area of 33 square inches, what is the area of each small opening?

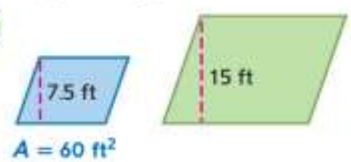
Practice and Problem Solving

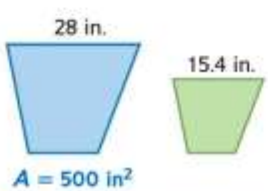
Extra Practice is on page R11.

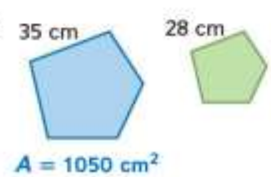
Example 1 TEKS G.10(B)

For each pair of similar figures, find the area of the green figure.

6\. 

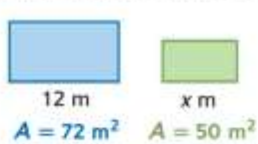
7\. 

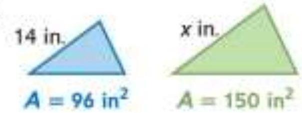
8\. 

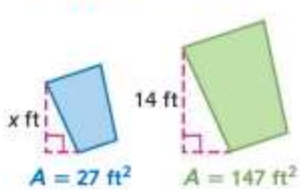
9\. 

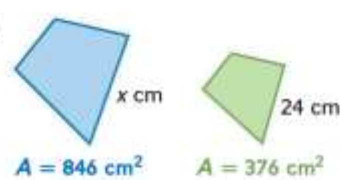
Example 2 TEKS G.10(B)

MP **ANALYZE RELATIONSHIPS** **For each pair of similar figures, use the given areas to find the scale factor of the blue to the green figure. Then find x.**

10\. 

11\. 

12\. 

13\. 

Example 3
TEKS G.10(B)

14. CRAFTS Marina crafts unique trivets and other kitchenware. Her basic trivet design is an equilateral triangle with an area of about 3.9 square inches. She plans to make trivet A by increasing each side by $\frac{4}{3}$. To make trivet B, Marina will double the length of one side while keeping the height as measured from the doubled side the same as the basic trivet. What are the approximate areas of trivets A and B?

15. CHANGING DIMENSIONS A circle has a radius of 24 inches.

a. If the area is doubled, how does the radius change?

b. How does the radius change if the area is tripled?

c. What is the change in the radius if the area is increased by a factor of x?

16. CHANGING DIMENSIONS A polygon has an area of 144 square meters.

a. If the area is doubled, how does each side length change?

b. How does each side length change if the area is tripled?

c. What is the change in each side length if the area is increased by a factor of x?

17. BAKING Kaitlyn wants to use one of two regular hexagonal cake pans for a recipe she is making. The side length of the larger pan is 4.5 inches, and the area of the base of the smaller pan is 41.6 square inches.

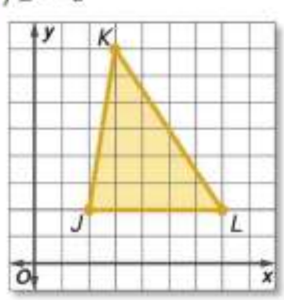
a. What is the side length of the smaller pan?

b. The recipe that Kaitlyn is using calls for a circular cake pan with an 8-inch diameter. Which pan should she choose? Explain your reasoning.

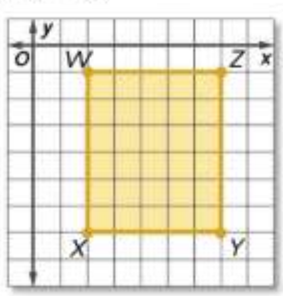
18. MP APPLY MATH Federico's family is putting hardwood floors in the two geometrically similar rooms shown. If the cost of flooring is constant and the flooring for the kitchen cost $2000, what will be the total flooring cost for the two rooms? Round to the nearest hundred dollars.

COORDINATE GEOMETRY Find the area of each figure. Use the segment length given to find the area of a similar polygon.

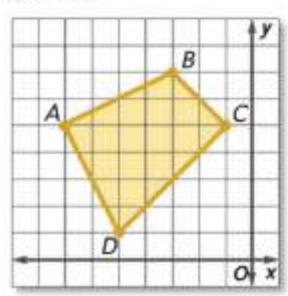
19 $J'L' = 3$



20. $W'X' = 8$



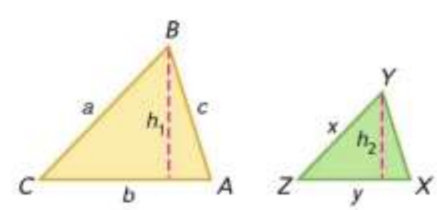
21. $B'C' = 5$



22. PROOF Write a paragraph proof.

Given: $\triangle ABC \sim \triangle XYZ$

Prove: $\frac{\text{area of } \triangle ABC}{\text{area of } \triangle XYZ} = \frac{a^2}{x^2}$



23. **STATISTICS** The graph shows the increase in high school tennis participation from 2000 to 2010.

a. Explain why the graph is misleading.

b. How could the graph be changed to more accurately represent the growth in high school tennis participation?

24. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate changing dimensions proportionally in three-dimensional figures.

a. **Tabular** Copy and complete the table below for each scale factor of a rectangular prism that is 2 inches by 3 inches by 5 inches.

Scale Factor	Length (in.)	Width (in.)	Height (in.)	Volume (in^3)	Ratio of Scaled Volume to Initial Volume
1	3	2	5		
2					
3					
4					
5					
10					

b. **Verbal** Make a conjecture about the relationship between the scale factor and the ratio of the scaled volume to the initial volume.

c. **Graphical** Make a scatter plot of the scale factor and the ratio of the scaled volume to the initial volume using the **STAT PLOT** feature on your graphing calculator. Then use the **STAT CALC** feature to approximate the function represented by the graph.

d. **Algebraic** Write an algebraic expression for the ratio of the scaled volume to the initial volume in terms of scale factor k.

TEKS G.10(B)

H.O.T. Problems Use Higher-Order Thinking Skills

25. **ERROR ANALYSIS** Violeta and Gavin are trying to come up with a formula that can be used to find the area of a circle with a radius r after it has been enlarged by a scale factor k. Is either of them correct? Explain your reasoning.

Violeta	Gavin
$A = k\pi r^2$	$A = \pi(r^2)^k$

26. **PROBLEM SOLVING** If you want the area of a polygon to be x% of its original area, by what scale factor should you multiply each side length?

27. **ANALYZE RELATIONSHIPS** A regular n-gon is enlarged, and the ratio of the area of the enlarged figure to the area of the original figure is R. Write an equation relating the perimeter of the enlarged figure to the perimeter of the original figure Q.

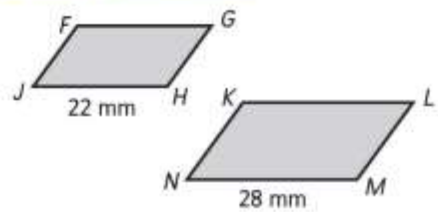
28. **ORGANIZE IDEAS** Draw a pair of similar figures with areas that have a ratio of 4:1. Explain.

29. **WRITING IN MATH** Explain how to find the area of an enlarged polygon if you know the area of the original polygon and the scale factor of the enlargement.

Example

TEKS G.10(B) MP G.1(E), G.1(F)

TEKS REVIEW Alisha uses geometry software to draw ▱*FGHJ*, as shown. According to the software, the area of ▱*FGHJ* is 200 square millimeters. Alisha also uses the software to create ▱*KLMN* so that ▱*FGHJ* ~ ▱*KLMN*. Which of the following is closest to the area of ▱*KLMN*?



A 123 mm^2

B 157 mm^2

C 255 mm^2

D 324 mm^2

It is given that ▱*FGHJ* ~ ▱*KLMN* and the area of ▱*FGHJ* is 200 square millimeters. From the figure, $HJ = 22$ millimeters and $MN = 28$ millimeters. From the similarity statement, HJ and MN are corresponding sides.

Since the two parallelograms are similar, their areas are proportional to the square of the scale factor between them. Use a proportion to solve for the area of ▱*KLMN*.

$$\frac{\text{area of ▱}KLMN}{\text{area of ▱}FGHJ} = \left(\frac{28}{22}\right)^2$$ Write a proportion.

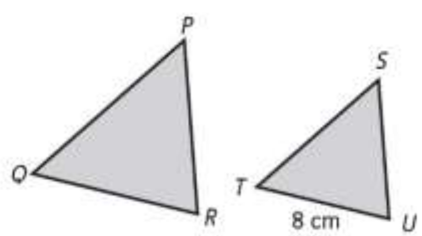
$$\frac{\text{area of ▱}KLMN}{200} = \frac{784}{484}$$ Area of ▱$FGHJ = 200$ and $\left(\frac{28}{22}\right)^2 = \frac{784}{484}$

$$\text{area of ▱}KLMN = \frac{784}{484} \cdot 200$$ Multiply each side by 200.

$$\text{area of ▱}KLMN \approx 323.967$$ Use a calculator.

Among the given choices, 324 mm^2 is closest to the area. The correct answer is choice D.

30. **ACT/SAT** In the figure, $\triangle PQR \sim \triangle STU$. The area of $\triangle PQR$ is 50 square centimeters and the area of $\triangle STU$ is 32 square centimeters.



What is the length of $\overline{QR}$?

A 5.12 cm

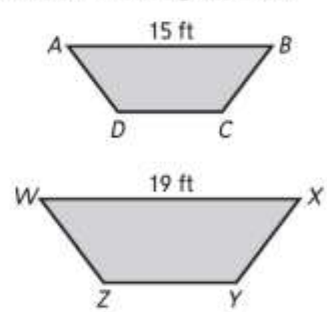
B 6.4 cm

C 10 cm

D 12.5 cm

E 26 cm

31. Connor drew the trapezoids below so that trapezoid *ABCD* ~ trapezoid *WXYZ*. The area of trapezoid *ABCD* is 55 square feet.



Which of the following is the best estimate of the area of trapezoid *WXYZ*? TEKS G.10(B) MP G.1(E), G.1(F)

F 88 ft^2

G 70 ft^2

H 43 ft^2

J 34 ft^2

CHAPTER 11

Study Guide and Review

Go Online! for Vocabulary Review Games and key vocabulary in 13 languages

Study Guide

KeyConcepts

Areas of Parallelograms and Triangles (Lesson 11-1)

- The area A of a parallelogram is the product of a base b and its corresponding height h. $A = bh$
- The area A of a triangle is one half the product of a base b and its corresponding height h. $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

Areas of Trapezoids, Rhombi, and Kites (Lesson 11-2)

- The area A of a trapezoid is one half the product of the height h and the sum of its bases, b_1 and b_2.
  $A = \frac{1}{2}h(b_1 + b_2)$
- The area A of a rhombus or kite is one half the product of the lengths of its diagonals, d_1 and d_2.
  $A = \frac{1}{2}d_1d_2$

Areas of Circles and Sectors (Lesson 11-3)

- The area A of a circle is equal to π times the square of the radius r. $A = \pi r^2$
- The ratio of the area A of a sector to the area of the whole circle, πr^2, is equal to the ratio of the degree measure of the intercepted arc x to 360.
  Proportion: $\frac{A}{\pi r^2} = \frac{x}{360}$ Equation: $A = \frac{x}{360} \cdot \pi r^2$

Areas of Regular Polygons and Composite Figures (Lesson 11-4)

- The area A of a regular n-gon with side length s is one half the product of the apothem a and perimeter P.
  $A = \frac{1}{2}a(ns)$ or $A = \frac{1}{2}aP$

Areas of Similar Figures (Lesson 11-5)

- If two polygons are similar, then their areas are proportional to the square of the scale factor between them.
  If $ABCD \sim FGHJ$, then $\frac{\text{area of } FGHJ}{\text{area of } ABCD} = \left(\frac{FG}{AB}\right)^2$.

FOLDABLES® StudyOrganizer

Use your Foldable to review the chapter. Working with a partner can be helpful. Ask for clarification of concepts as needed.

KeyVocabulary

apothem (p. 807)

base of a parallelogram (p. 779)

base of a triangle (p. 781)

center of a regular polygon (p. 807)

central angle of a regular polygon (p. 807)

composite figure (p. 809)

height of a parallelogram (p. 779)

height of a trapezoid (p. 789)

height of a triangle (p. 781)

radius of a regular polygon (p. 807)

sector of a circle (p. 799)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The center of a trapezoid is the perpendicular distance between the bases.
2. A slice of pizza is a sector of a circle.
3. The center of a regular polygon is the distance from the middle to the circle circumscribed around the polygon.
4. The segment from the center of a square to the corner can be called the radius of the square.
5. A segment drawn perpendicular to a side of a regular polygon is called an apothem of the polygon.
6. The measure of each radial angle of a regular n-gon is $\frac{360}{n}$.
7. The apothem of a polygon is the perpendicular distance between any two parallel bases.
8. The height of a triangle is the length of an altitude drawn to a given base.
9. Any side of a parallelogram can be called the height of a parallelogram.
10. The center of a regular polygon is also the center of its circumscribed circle.

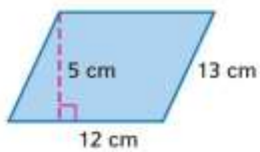
Lesson-by-Lesson Review

11-1 Areas of Parallelograms and Triangles

TEKS G.10(B)

Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.

11.



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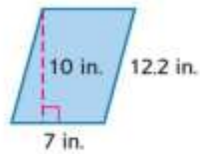

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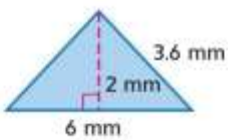
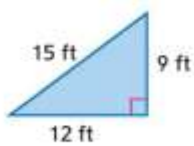


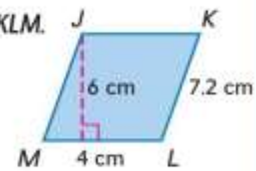
14.



15. **PAINTING** Two of the walls of an attic in an A-frame house are triangular, each with a height of 12 feet and a width of 22 feet. How much paint is needed to paint one end of the attic?

Example 1

Find the perimeter and area of ▱JKLM.



Perimeter

Perimeter of ▱JKLM $= JK + KL + LM + JM$

$= 4 + 7.2 + 4 + 7.2$ or 22.4 cm

Area

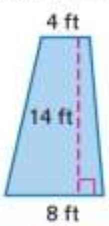
$A = bh$ — Area of a parallelogram

$= (4)(6)$ or 24 cm^2 — $b = 4$ and $h = 6$

11-2 Areas of Trapezoids, Rhombi, and Kites

Find the area of each trapezoid, rhombus, or kite.

16.



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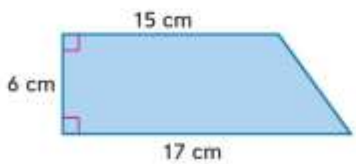

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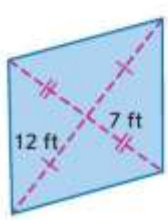
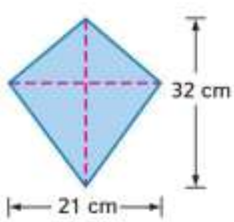


19.

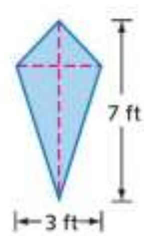


20. **KITES** Team Dragon's kite is 4 feet long and 3 feet across. How much fabric does it take to make their kite?

Example 2

Find the area of each rhombus or kite.

a.

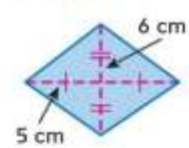


$A = \frac{1}{2}d_1d_2$ — Area of a kite

$= \frac{1}{2}(7)(3)$ — $d_1 = 7$ and $d_2 = 3$

$= 10.5\text{ ft}^2$ — Simplify.

b.



Since the diagonals of a rhombus bisect each other, the lengths of the diagonals are $6 + 6$ or 12 centimeters and $5 + 5$ or 10 centimeters.

$A = \frac{1}{2}d_1d_2$ — Area of a rhombus

$= \frac{1}{2}(10)(12)$ — $d_1 = 10$ and $d_2 = 12$

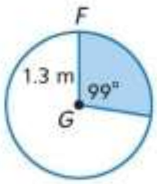
$= 60\text{ cm}^2$ — Simplify.

11-3 Areas of Circles and Sectors

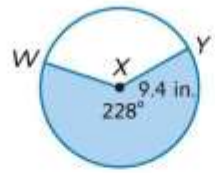
TEKS G.11(B), G.12(C)

Find the area of each shaded sector. Round to the nearest tenth.

21.



22.



23. **BICYCLES** A bicycle tire decoration covers $\frac{1}{9}$ of the circle formed by the tire. If the tire has a diameter of 26 inches, what is the area of the decoration?

24. **PIZZA** Charlie and Kris ordered a 16-inch pizza and cut the pizza into 12 slices.

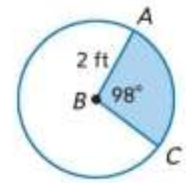
   a. If Charlie ate 3 pieces, what area of the pizza did he eat?

   b. If Kris ate 2 pieces, what area of the pizza did she eat?

   c. What is the area of leftover pizza?

Example 3

Find the area of the shaded sector. Round to the nearest tenth.



$A = \frac{x}{360} \cdot \pi r^2$ Area of a sector

$= \frac{98}{360} \cdot \pi(2)^2$ Substitution

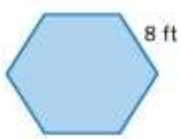
$\approx 3.4 \text{ ft}^2$ Simplify.

11-4 Areas of Regular Polygons and Composite Figures

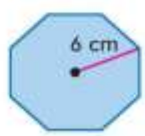
TEKS G.11(A), G.11(B)

Find the area of each regular polygon or composite figure. Round to the nearest tenth.

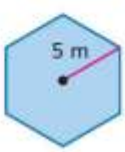
25.



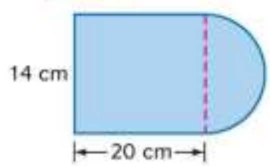
26.



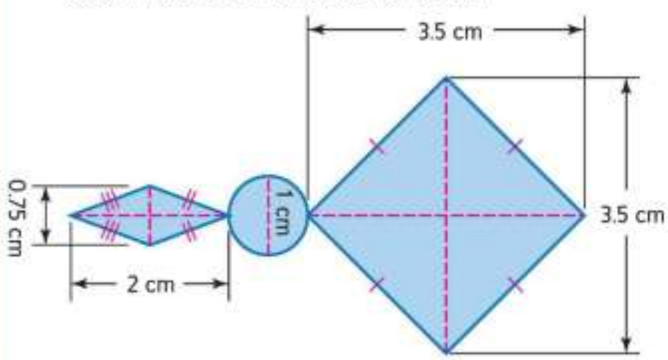
27.



28.

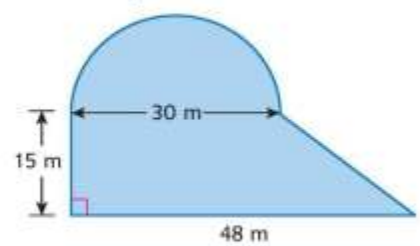


29. **JEWELRY** What is the area of the pendant shown below? Round to the nearest hundredth.



Example 4

Find the area of the figure.



The composite shape is made up of a semicircle and a trapezoid.

Area = Area of semicircle + Area of trapezoid

$= \frac{180}{360} \cdot \pi \cdot r^2 + \frac{1}{2} \cdot h \cdot (b_1 + b_1)$

$\approx \frac{180}{360} \cdot \pi \cdot 15^2 + \frac{1}{2} \cdot 15 \cdot (30 + 48)$

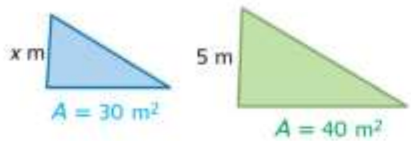
$\approx 112.5\pi + 585$ or about 938.4 m^2

11-5 Areas of Similar Figures

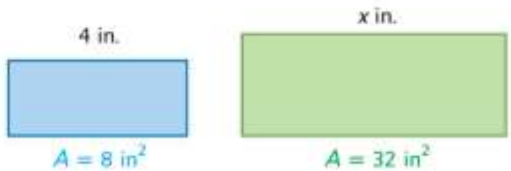
TEKS G.10(B)

For each pair of similar figures, use the given areas to find the scale factor from the blue to the green figure. Then find x.

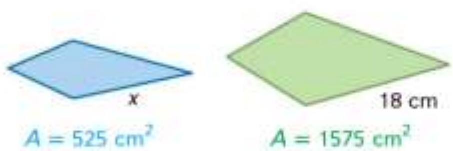
30.



31.

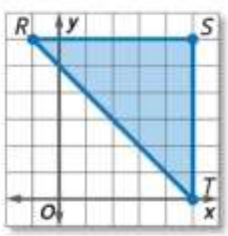


32.

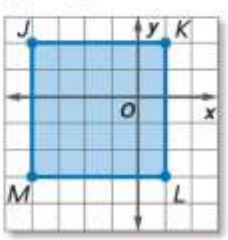


COORDINATE GEOMETRY Find the area of each figure. Use the segment length given to find the area of a similar polygon.

33. $R'S' = 3$



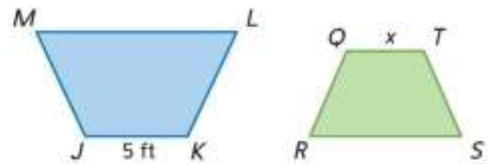
34. $K'L' = 15$



35. **LAND OWNERSHIP** Joshua's land is 600 square miles. He purchases an additional plot that is half the length and one-fourth the width of his original plot. What is the new area of his land?

Example 5

The area of trapezoid $JKLM$ is 138 square feet. The area of trapezoid $QRST$ is 5.52 square feet. If trapezoid $JKLM \sim$ trapezoid $QRST$, find the scale factor from trapezoid $JKLM$ to trapezoid $QRST$ and the value of x.



Let k be the scale factor between trapezoid $JKLM$ and trapezoid $QRST$.

$$\frac{\text{Area of trapezoid } JKLM}{\text{Area of trapezoid } QRST} = k^2$$ Theorem 11.1

$$\frac{138}{5.52} = k^2$$ Substitution

$$5 = k$$ Take the positive square root of each side.

So, the scale factor from trapezoid $JKLM$ to trapezoid $QRST$ is 5. Use this scale factor to find the value of x.

$$\frac{JK}{QT} = k$$ The ratio of corresponding lengths of similar polygons is equal to the scale factor between the polygons.

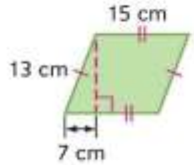
$$\frac{5}{x} = 5$$ Substitution

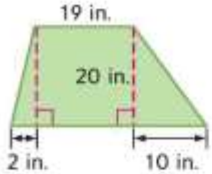
$$1 = x$$ Simplify.

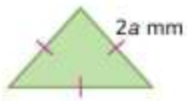
CHAPTER 11 Practice Test

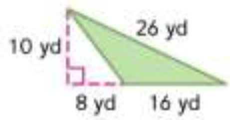
Go Online! for another Chapter Test

Find the area and perimeter of each figure. Round to the nearest tenth if necessary.

1. 

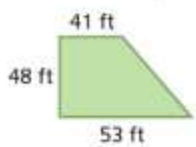
2. 

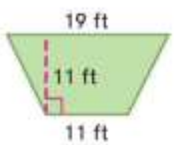
3. 

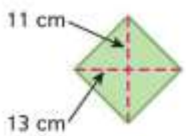
4. 

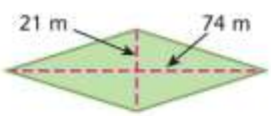
5. **ARCHAELOGY** The tile pattern shown was used in Pompeii for paving. If the diagonals of each rhombus are 2 and 3 inches, what area makes up each "cube" in the pattern?

Find the area of each figure. Round to the nearest tenth if necessary.

6. 

7. 

8. 

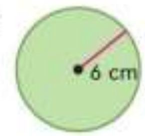
9. 

10. **GEMOLOGY** A gem is cut in a kite shape. It is 6.2 millimeters wide at its widest point and 5 millimeters long. What is the area?

11. **ALGEBRA** The area of a triangle is 16 square units. The base of the triangle is $x + 4$ and the height is x. Find x.

12. **ASTRONOMY** A large planetarium in the shape of a dome is being built. When it is complete, the base of the dome will have a circumference of 870 meters. How many square meters of land were required for this planetarium?

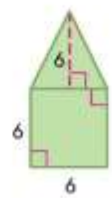
Find the area of each circle or sector. Round to the nearest tenth.

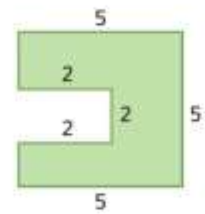
13. 

14. 

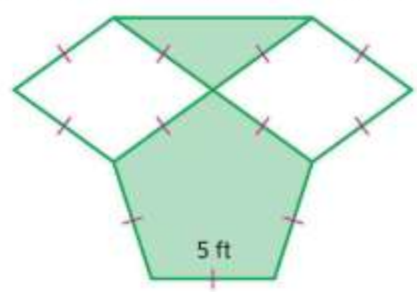
15. **MURALS** An artisan is creating a circular street mural for an art festival. The mural is going to be 50 feet wide. One sector of the mural spans 38°. What is the area of this sector to the nearest square foot?

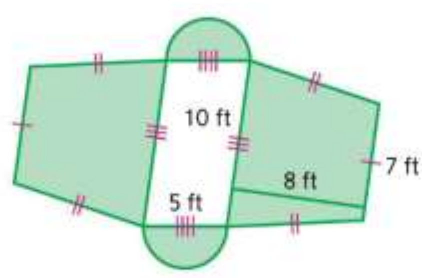
Find the perimeter and area of each figure. Round to the nearest tenth if necessary.

16. 

17. 

FLOORING **Brian's service project is to build tree houses in the city park. Find the shaded area in each treehouse. Round to the nearest tenth.**

18. 

19. 

20. **BAKING** Todd wants to make a cheesecake for a birthday party. The recipe calls for a 9-inch diameter round pan. Todd only has square pans. He has an 8-inch square pan, a 9-inch square pan, and a 10-inch square pan. Which pan comes closest in area to the one that the recipe suggests?

Preparing for Assessment
Test-Taking Strategies

Solve Multi-Step Problems

Some problems that you will encounter on standardized tests require you to solve multiple parts in order to come up with the final solution. Use this lesson to practice these types of problems.

Strategies for Solving Multi-Step Problems

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve? What information is given?
- Are there any intermediate steps that need to be completed before I can solve the problem?

Step 2

Organize your approach.

- List the steps you will need to complete in order to solve the problem.
- Remember that there may be more than one possible way to solve the problem.

Step 3

Solve and check.

- Work as efficiently as possible to complete each step and solve.
- If time permits, check your answer.

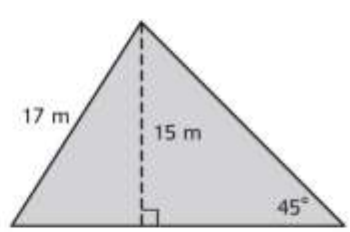
Texas Assessment Example TEKS G.9(B) MP G.1(E)

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

What is the area of the triangle? Round your answer to the nearest tenth if necessary.

A $112.5\ m^2$ C $152.5\ m^2$

B $172.5\ m^2$ D $195.5\ m^2$

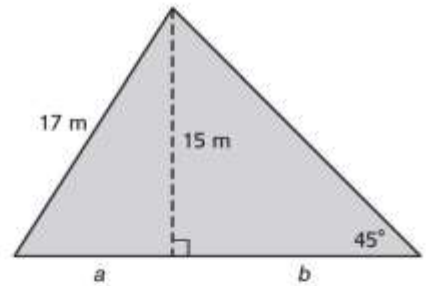


Read the problem statement and study the figure carefully. At first glance, the problem may appear fairly straightforward. Notice, however, that you must first find the base of the triangle before you can find its area. Organize an approach to solve the problem.

Step 1 Use the Pythagorean Theorem to find a.

Step 2 Use trigonometry to find b.

Step 3 Find the area of the triangle.



Step 1 Find a.

$a^2 + 15^2 = 17^2$

$a^2 = 289 - 225$

$a^2 = 64$

$a = 8$

Step 2 Find b. This is a 45°-45°-90° triangle. So $b = 15$.

Step 3 Find the area of the triangle.

The base of the triangle is $a + b$ or 23 meters.

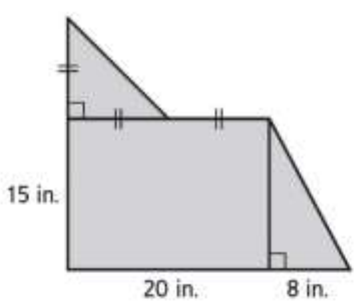
$A \approx \frac{1}{2}(23)(15)$ or 172.5

So, the area of the triangle is about 172.5 square meters. The answer is B.

Exercises

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

1. Which of the following best represents the area of the figure shown? TEKS G.11(B) MP G.1(E)



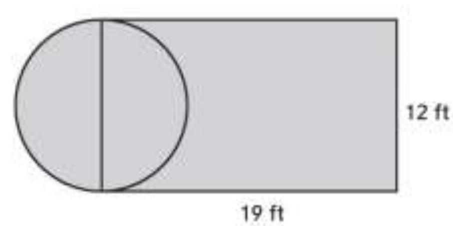
A 350 in^2

B 410 in^2

C 460 in^2

D 470 in^2

2. Kaleb is painting the basketball key shown below. How much area will he need to cover? Round to the nearest tenth. TEKS G.11(B) MP G.1(E)



F 113.1 ft^2

G 228.0 ft^2

H 284.5 ft^2

J 341.1 ft^2

CHAPTER 11

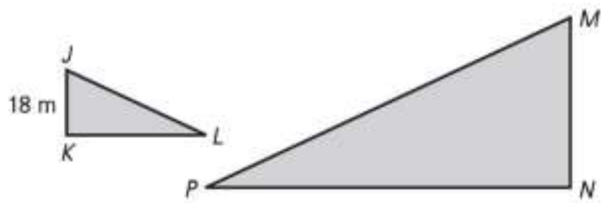
Preparing for Assessment

Cumulative Review

Go Online! for Standardized Test Practice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. In the figure, $\triangle JKL \sim \triangle MNP$. The area of $\triangle JKL$ is 324 square meters and the area of $\triangle MNP$ is 1764 square meters.



Which of the following is closest to the length of $\overline{MN}$? TEKS G.10(B) MP G.1(E)

A 3 m

B 8 m

C 42 m

D 98 m

2. Which of the following is a true statement about the circle with equation $x^2 + y^2 - 6x + 6y = -14$? TEKS G.12(E) MP G.1(G)

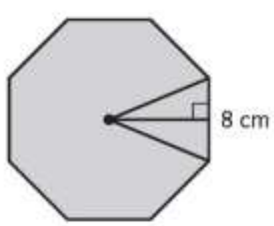
F The circle lies entirely in Quadrant II.

G The radius of the circle is 4.

H The circle intersects both axes.

J The center of the circle is $(3, -3)$.

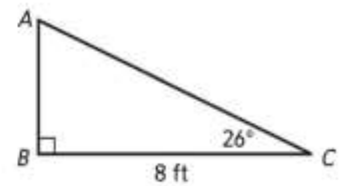
3. Which of the following is the best estimate of the area of the regular octagon shown below? TEKS G.11(A) MP G.1(C)



A 618 cm^2 C 128 cm^2

B 309 cm^2 D 39 cm^2

4. Malia wants to put a fence around the three sides of a right-triangular plot in her garden. She measures one side length and one acute angle, as shown in the figure.



Which of the following is the best estimate of the length of the fence she will need? TEKS G.9(A) MP G.1(A)

F 33.3 ft H 20.8 ft

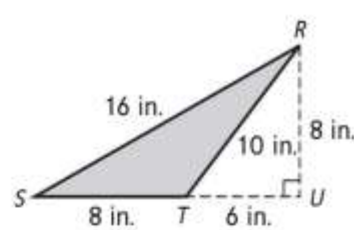
G 30.1 ft J 19.1 ft

5. **GRIDDABLE** A circular pizza has a diameter of 18 inches. Charles slices the pizza into 12 equal sectors. What is the area of each sector, in square inches? Round to the nearest tenth. TEKS G.12(C) MP G.1(A)

Test-Taking Tip

Question 5 There are two ways to solve this problem. You can determine the measure of the intercepted arc for each sector and then use the formula for the area of a sector, or you can find the area of the whole pizza and divide by 12.

6. Kento enlarges $\triangle RST$ so that each dimension is 4 times the corresponding dimension shown in the figure.

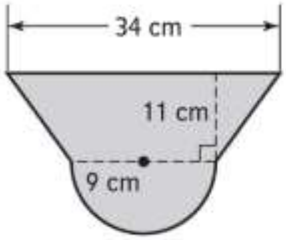


What is the area of the enlarged triangle? TEKS G.10(B) MP G.1(F)

A 128 in^2 C 640 in^2

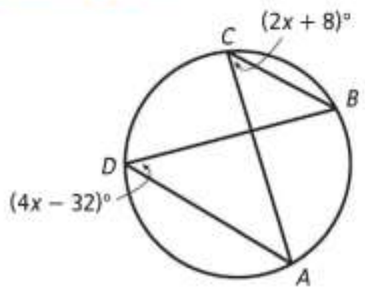
B 512 in^2 D 896 in^2

7. Which of the following is the best estimate of the area of the composite figure shown here? TEKS G.11(B) MP G.1(F)

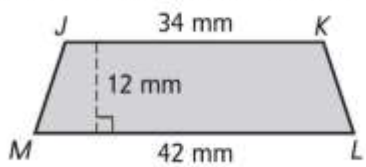


F $540\ cm^2$ | H $364\ cm^2$
G $413\ cm^2$ | J $286\ cm^2$

8. **GRIDDABLE** What is the measure of $\widehat{AB}$ in degrees? TEKS G.12(A) MP G.1(F)



9. Dayana drew the trapezoid shown here.



Which of the following will allow Dayana to create a new trapezoid with half the area of trapezoid *JKLM*? TEKS G.10(B) MP G.1(G)

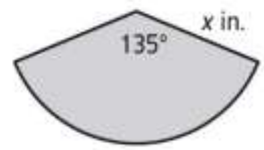
I. Draw a trapezoid in which each base is half as long as in trapezoid *JKLM*.
II. Draw a trapezoid in which the height is half as long as in trapezoid *JKLM*.
III. Draw a trapezoid in which every dimension is half the corresponding dimension in trapezoid *JKLM*.

A I only | C III only
B II only | D I and II only

10. Zoe plots the point $P(-1, 2)$. Then she reflects it in the line $y = x$ and dilates the image using a dilation centered at the origin with a scale factor of 0.5. What are the coordinates of the final image? TEKS G.3(B) MP G.1(C)

F $(1, -0.5)$ | H $(-0.5, -1)$
G $(0.5, 1)$ | J $(-0.5, 1)$

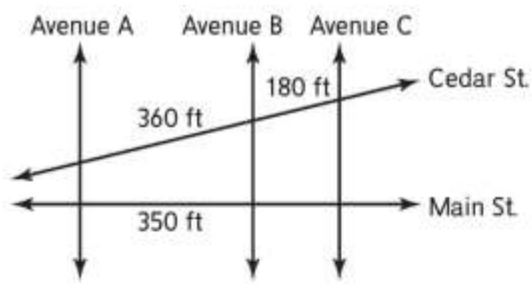
11. Marcello is making a cone-shaped base for a lamp. He goes online to order a piece of metal that is a sector of a circle, as shown. According to the Web site, the area of the sector is 180 square inches.



Which of the following is the best estimate of the value of x? TEKS G.12(C) MP G.1(A)

A 4.6 | C 12.4
B 7.6 | D 76.4

12. The figure shows a map of the streets in downtown Oakville. Avenues A, B, and C are all parallel.



What is the distance along Main Street from Avenue A to Avenue C? TEKS G.8(A) MP G.1(A)

F 175 ft | H 525 ft
G 520 ft | J 1050 ft

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	11-5	10-8	11-4	8-4	11-3	11-1	11-4	10-4	11-2	9-6	11-3	7-4